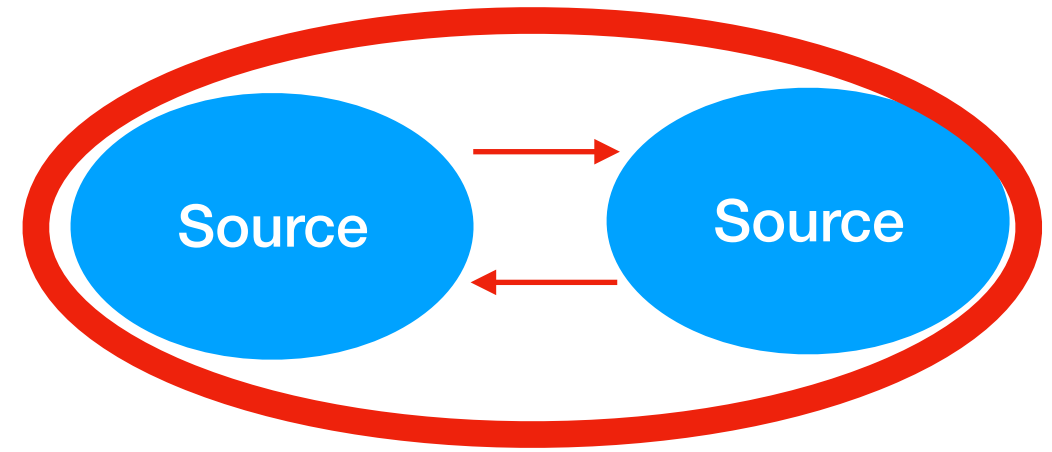
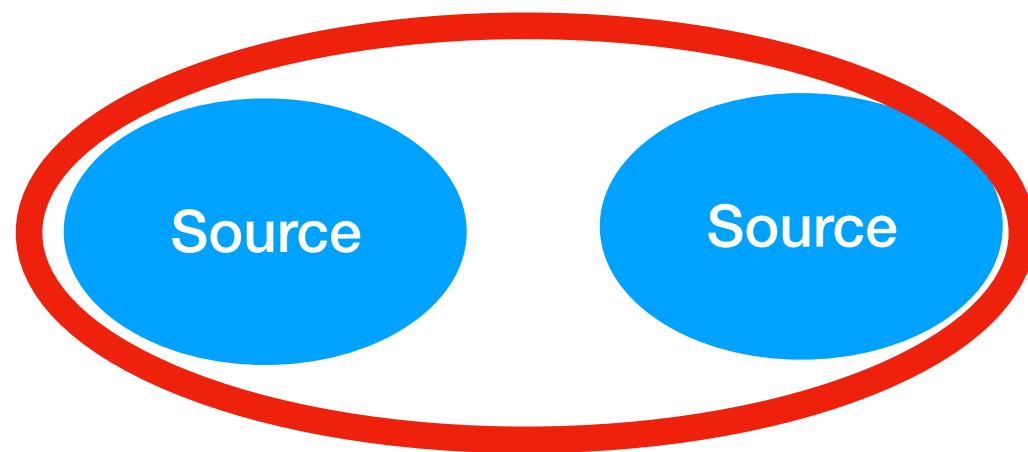
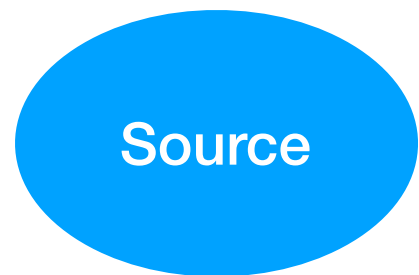


Migration entre 2 sites en environnement constant

(qui ne dépend pas du temps)

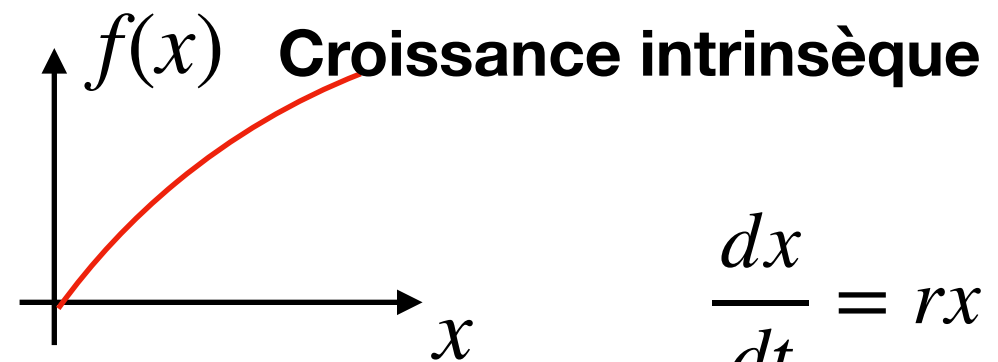


Que comparer ?

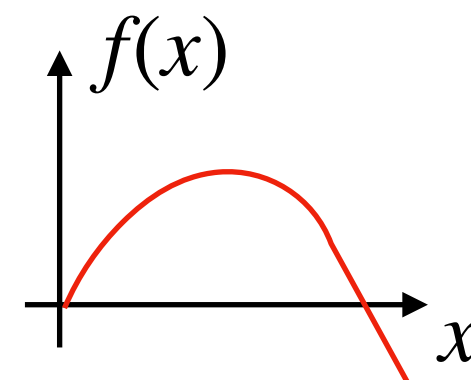


Dynamique sur 1 source ?

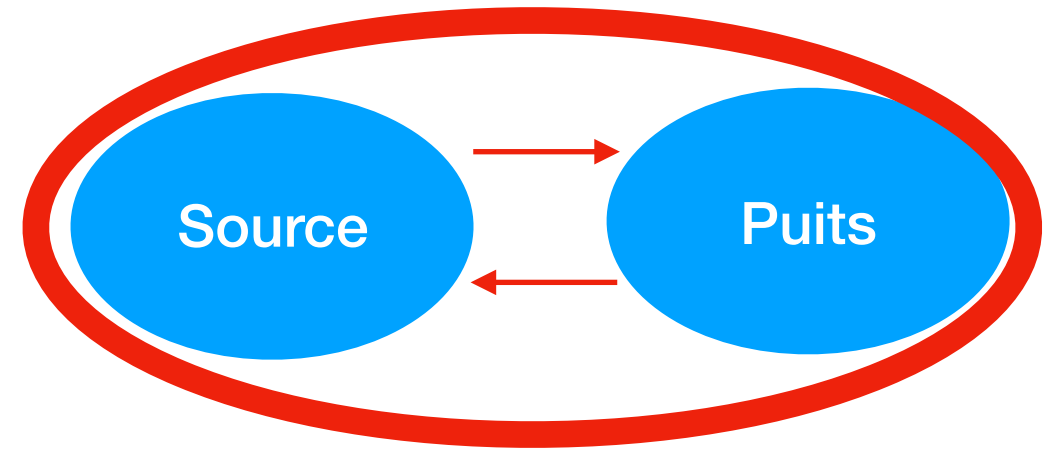
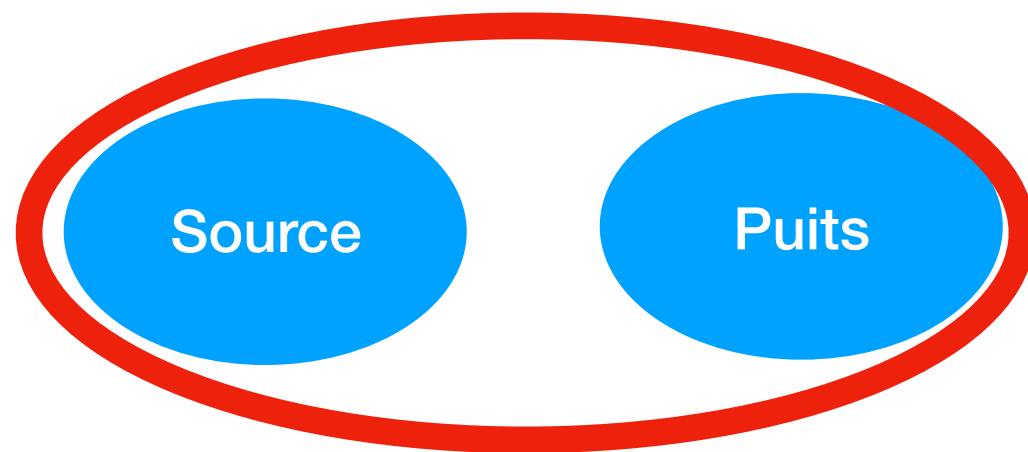
$$\frac{dx}{dt} = f(x)$$



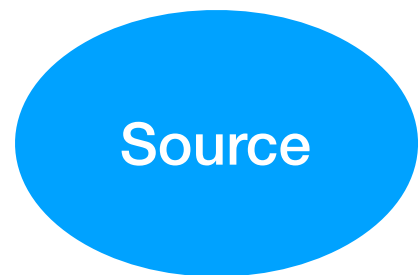
$$\frac{dx}{dt} = rx$$



$$\frac{dx}{dt} = rx \left(1 - \frac{x}{K} \right)$$

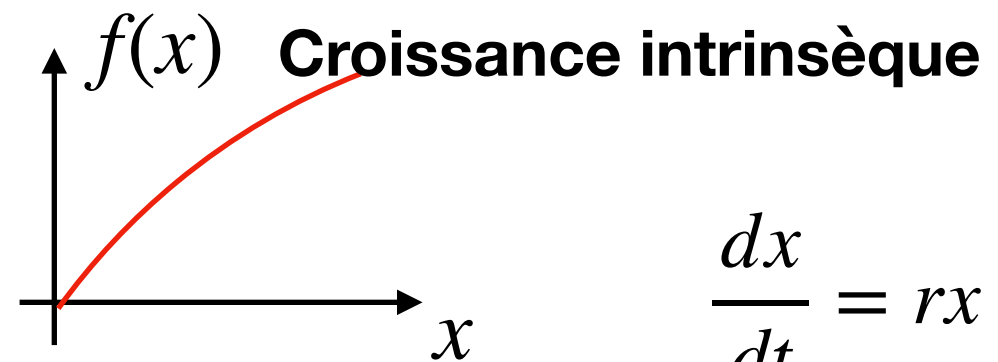


Que comparer ?



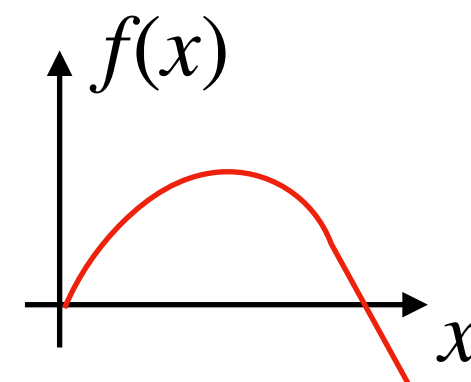
Dynamique sur 1 source ?

$$\frac{dx}{dt} = f(x)$$



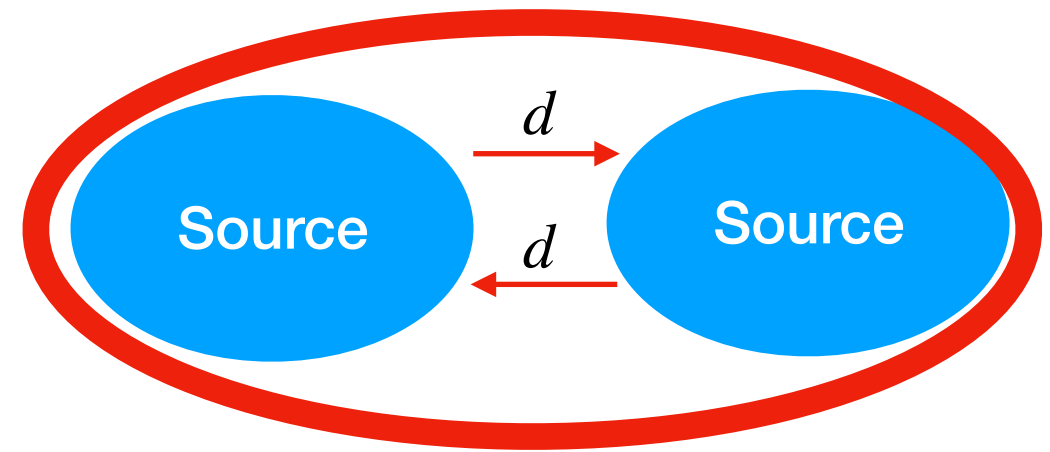
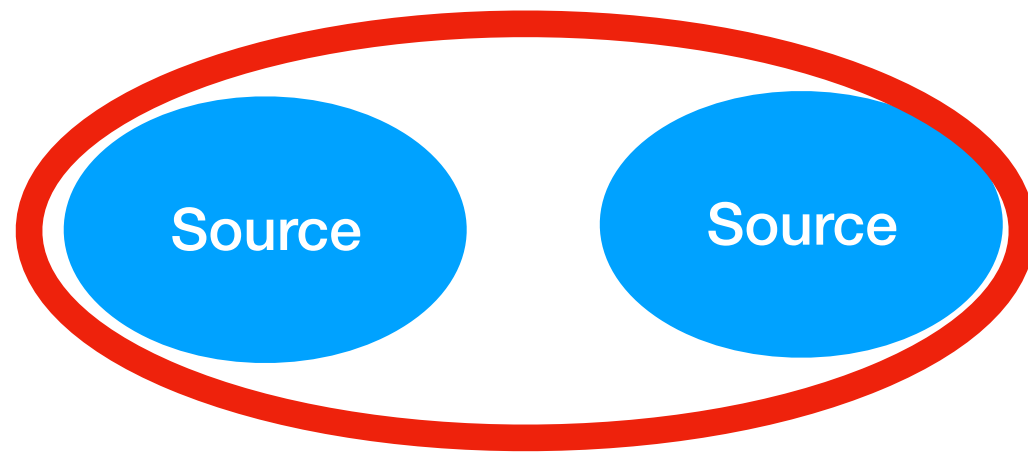
Croissance intrinsèque

$$\frac{dx}{dt} = rx$$



$$\frac{dx}{dt} = rx \left(1 - \frac{x}{K} \right)$$

Total Equilibrium Population



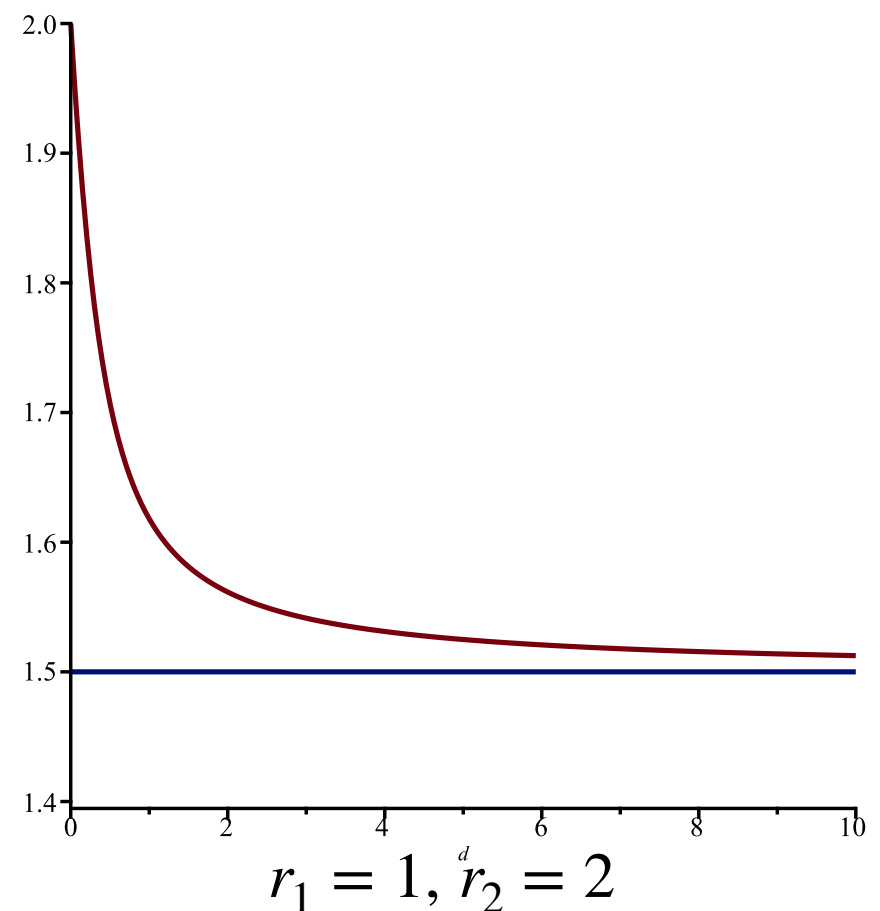
Comparer les taux de croissance

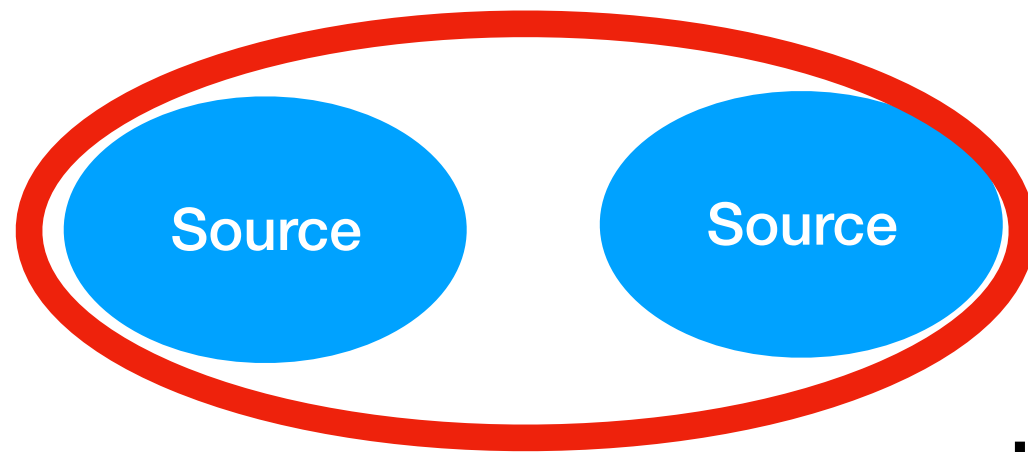
$$\frac{r_1}{2} + \frac{r_2}{2} - d + \frac{\sqrt{4d^2 + r_1^2 - 2r_1r_2 + r_2^2}}{2}$$

$$\frac{dx_1}{dt} = r_1x_1 + d(x_2 - x_1)$$

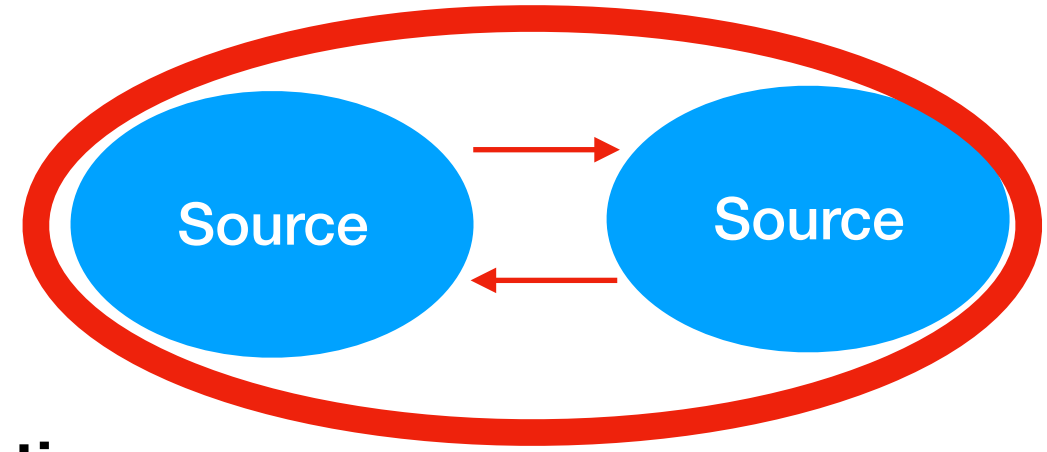
$$\frac{dx_2}{dt} = r_2x_2 + d(x_1 - x_2)$$

**En fait c'est pareil
si les deux taux de croissance
ne sont pas de même signe :
(Source-Puits)**





Total Equilibrium Population



Logistique
Comparer les "TEP"

$$\begin{aligned}\frac{dx_1}{dt} &= rx_1 \left(1 - \frac{x_1}{K_1}\right) + d(x_2 - x_1) \\ \frac{dx_2}{dt} &= rx_2 \left(1 - \frac{x_2}{K_2}\right) + d(x_1 - x_2)\end{aligned}$$

$$\begin{aligned}0 &= rx_1^* \left(1 - \frac{x_1^*}{K_1}\right) + d(x_2^* - x_1^*) \\ 0 &= rx_2^* \left(1 - \frac{x_2^*}{K_2}\right) + d(x_1^* - x_2^*)\end{aligned}$$

$$d = 0 \implies K_1 + K_2$$

$$d > 0 \implies x_1^* + x_2^*$$

Migration entre 2 sites en environnement constant

$$\frac{dx}{dt} = \rho x(S - x)$$

$$rx \left(1 - \frac{x}{K} \right) = \frac{r}{K} x(K - x) = \rho x(S - x)$$

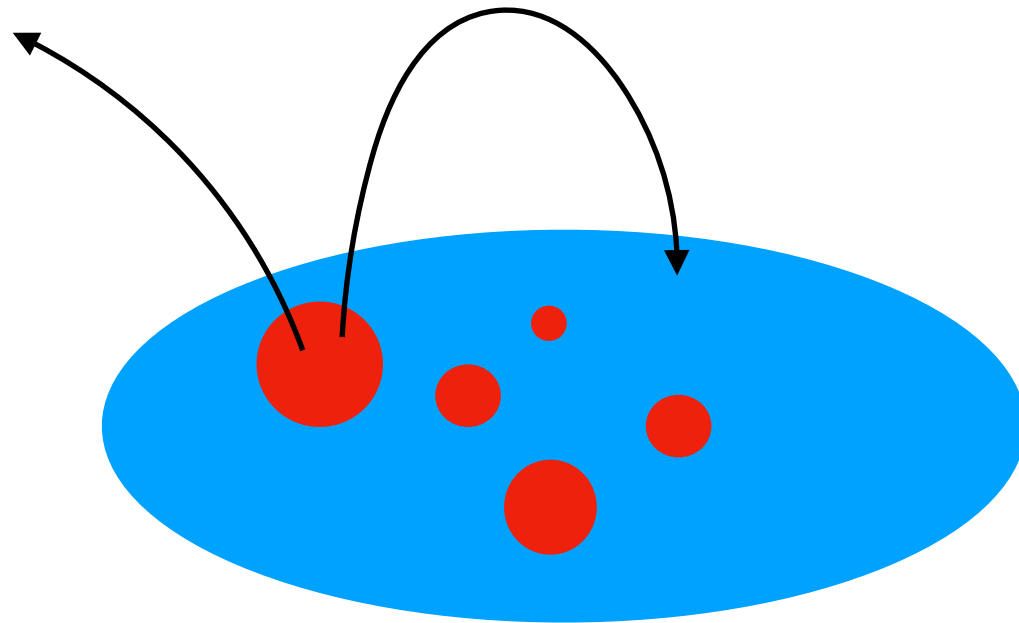
Logistique 1845



Comment justifier la pertinence de la logistique ?

Migration entre 2 sites en environnement constant

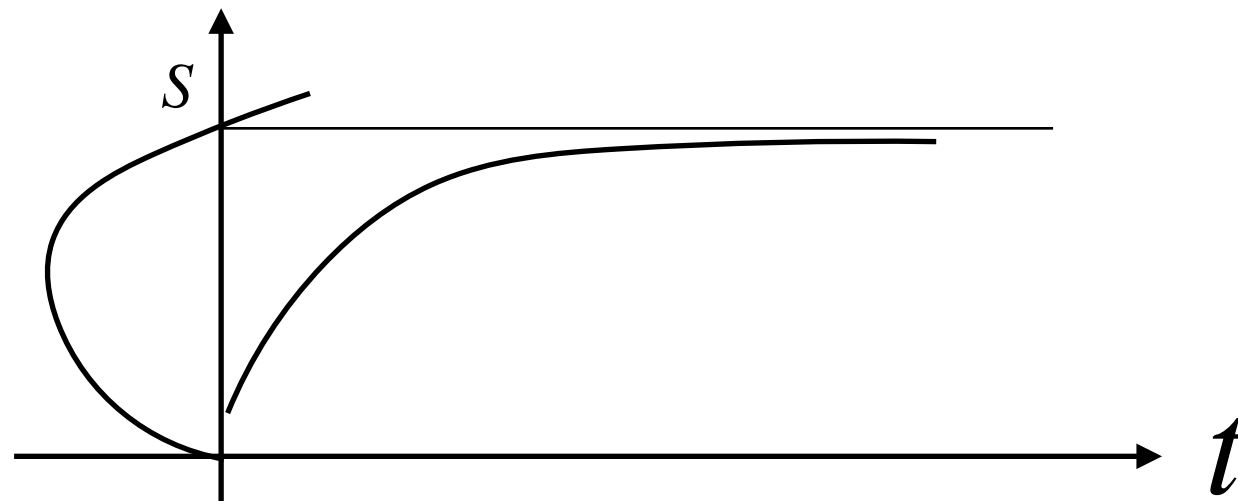
$$\frac{dx}{dt} = \rho x(S - x)$$



Une interprétation
en dynamique des populations

Rouge = surface colonisée = x

Bleu = surface colonisable = $S - x$

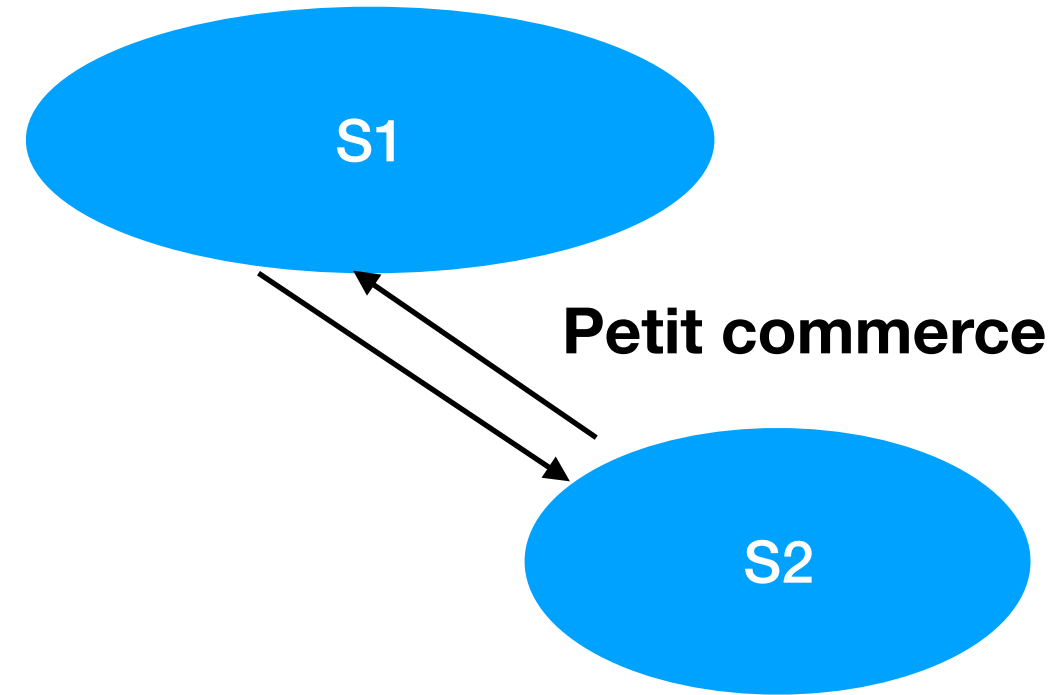


$$\frac{dx_1}{dt} = \rho x_1 (S_1 - x_1) + d(x_2 - x_1)$$

$$\frac{dx_2}{dt} = \rho x_2 (S_2 - x_2) + d(x_1 - x_2)$$

$$0 = \rho x_1^* (S_1 - x_1^*) + d(x_2^* - x_1^*)$$

$$0 = \rho x_2^* (S_2 - x_2^*) + d(x_1^* - x_2^*)$$



$$0 = (S_1 - x_1^*) + d \frac{(x_2^* - x_1^*)}{\rho x_1^*}$$

$$0 = (S_2 - x_2^*) + d \frac{(x_1^* - x_2^*)}{\rho x_2^*}$$

$$0 = (S_1 - x_1^*) + d \frac{(x_2^* - x_1^*)}{\rho x_1^*}$$

$$0 = (S_2 - x_2^*) + d \frac{(x_1^* - x_2^*)}{\rho x_2^*}$$

+

$$(S_1 + S_2) - (x_1^* + x_2^*) = \frac{d}{\rho} \left(\frac{(x_2^* - x_1^*)}{x_1^*} + \frac{(x_1^* - x_2^*)}{x_2^*} \right)$$

$$(S_1 + S_2) - (x_1^* + x_2^*) = \frac{d}{\rho} \left(\frac{(x_2^* - x_1^*)x_2^* + (x_1^* - x_2^*)x_1^*}{x_1^*x_2^*} \right)$$

$$(S_1 + S_2) - (x_1^* + x_2^*) = \frac{d}{\rho} \left(\frac{(x_2^* - x_1^*)^2}{x_1^*x_2^*} \right) > 0$$

Preuve mathématique que le libéralisme accroît la richesse des nations ?

$$0 = (S_1 - x_1^*) + d \frac{(x_2^* - x_1^*)}{\rho x_1^*}$$

$$0 = (S_2 - x_2^*) + d \frac{(x_1^* - x_2^*)}{\rho x_2^*}$$

+

$$(S_1 + S_2) - (x_1^* + x_2^*) = \frac{d}{\rho} \left(\frac{(x_2^* - x_1^*)}{x_1^*} + \frac{(x_1^* - x_2^*)}{x_2^*} \right)$$

$$(S_1 + S_2) - (x_1^* + x_2^*) = \frac{d}{\rho} \left(\frac{(x_2^* - x_1^*)x_2^* + (x_1^* - x_2^*)x_1^*}{x_1^*x_2^*} \right)$$

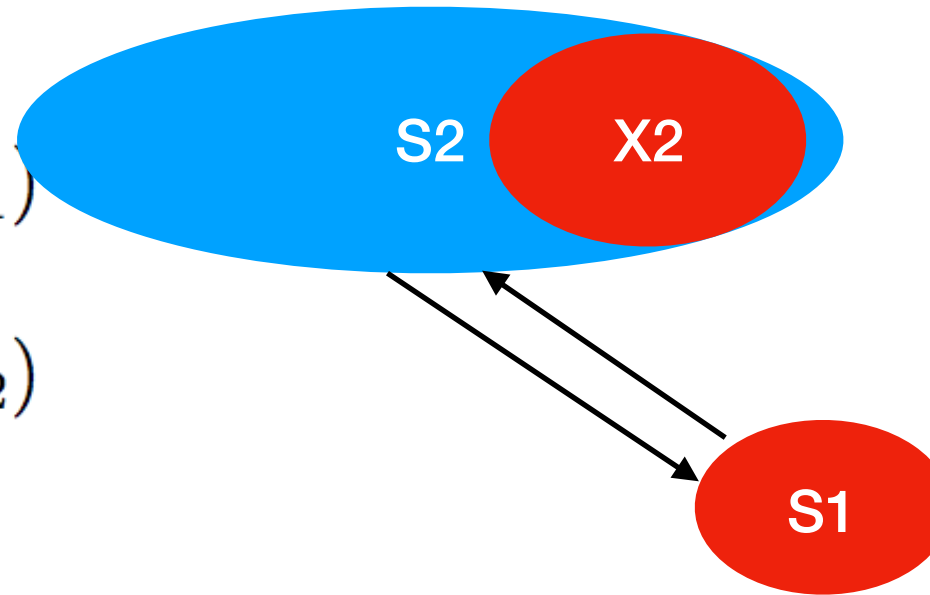
$$(S_1 + S_2) - (x_1^* + x_2^*) = \frac{d}{\rho} \left(\frac{(x_2^* - x_1^*)^2}{x_1^*x_2^*} \right)$$

Erreur de modélisation

$$n = (S_1 - x_1^*) + (S_2 - x_2^*) + d(x_2^* - x_1^*)$$

$$\frac{dx_1}{dt} = \rho x_1 (S_1 - x_1) + d(x_2 - x_1)$$

$$\frac{dx_2}{dt} = \rho x_2 (S_2 - x_2) + d(x_1 - x_2)$$



$$(S_1 + S_2) - (x_1^* + x_2^*) = \frac{d}{\rho} \left(\frac{(x_2^* - x_1^*)}{x_1^*} + \frac{(x_1^* - x_2^*)}{x_2^*} \right)$$

$$(S_1 + S_2) - (x_1^* + x_2^*) = \frac{d}{\rho} \left(\frac{(x_2^* - x_1^*)x_2^* + (x_1^* - x_2^*)x_1^*}{x_1^*x_2^*} \right)$$

$$(S_1 + S_2) - (x_1^* + x_2^*) = \frac{d}{\rho} \left(\frac{(x_2^* - x_1^*)^2}{x_1^*x_2^*} \right)$$

$$\frac{dx_1}{dt} = \rho x_1 (S_1 - x_1) + d(x_2 - x_1)$$

$$\frac{dx_2}{dt} = \rho x_2 (S_2 - x_2) + d(x_1 - x_2)$$

$$S_1 < S_2 \quad x_1 = S_1 \quad x_2 > x_1$$

$$\Rightarrow \frac{dx_1}{dt} > 0 \quad \text{Faux}$$

Erreur de modélisation

$$0 = (S_1 - x_1^*) + d \frac{(x_2^* - x_1^*)}{\rho x_1^*}$$

$$0 = (S_2 - x_2^*) + d \frac{(x_1^* - x_2^*)}{\rho x_2^*}$$

+

$$(S_1 + S_2) - (x_1^* + x_2^*) = \frac{d}{\rho} \left(\frac{(x_2^* - x_1^*)}{x_1^*} + \frac{(x_1^* - x_2^*)}{x_2^*} \right)$$

$$(S_1 + S_2) - (x_1^* + x_2^*) = \frac{d}{\rho} \left(\frac{(x_2^* - x_1^*)x_2^* + (x_1^* - x_2^*)x_1^*}{x_1^*x_2^*} \right)$$

$$(S_1 + S_2) - (x_1^* + x_2^*) = \frac{d}{\rho} \left(\frac{(x_2^* - x_1^*)^2}{x_1^*x_2^*} \right)$$

Erreur de modélisation

Mais valable pour l'interprétation

~~$$\frac{dx_1}{dt} = \rho x_1(S_1 - x_1) + d(x_2 - x_1)$$

$$\frac{dx_2}{dt} = \rho x_2(S_2 - x_2) + d(x_1 - x_2)$$~~

$$\frac{dx_1}{dt} = \rho x_1(S_1 - x_1) + d(\sigma_1(x_1)x_2 - x_1)$$

$$\frac{dx_2}{dt} = \rho x_2(S_2 - x_2) + d(\sigma_2(x_2)x_1 - x_2)$$

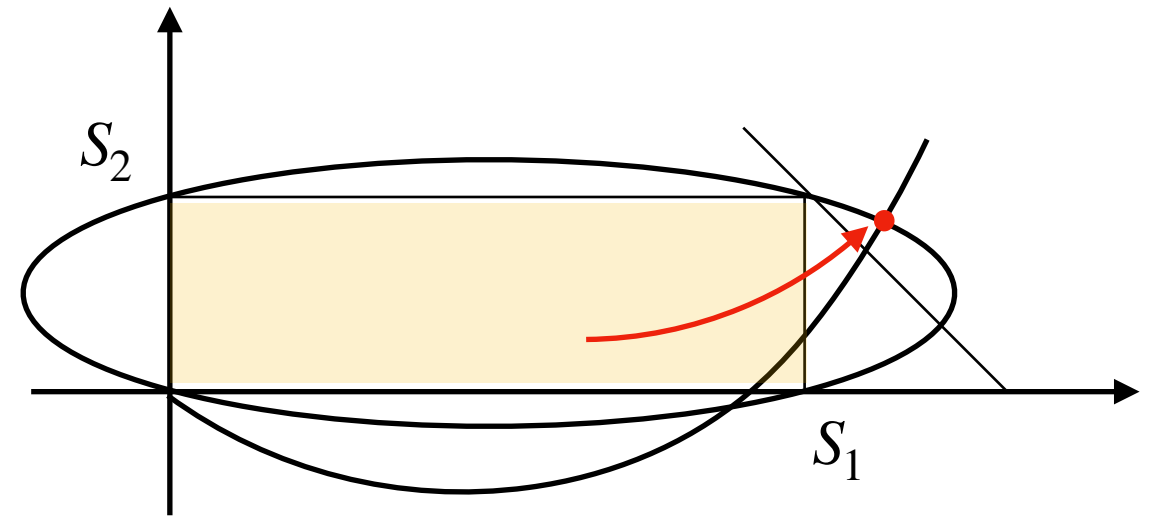
$$\sigma_i(x_i) = 1 \text{ si } x_i < S_i \text{ 0 sinon}$$

$$\frac{dx}{dt} = \rho x - \alpha x^2$$

$$0 = (S_1 - x_1^*) + d \frac{(x_2^* - x_1^*)}{\rho x_1^*}$$

$$0 = (S_2 - x_2^*) + d \frac{(x_1^* - x_2^*)}{\rho x_2^*}$$

+



$$(S_1 + S_2) - (x_1^* + x_2^*) = \frac{d}{\rho} \left(\frac{(x_2^* - x_1^*)}{x_1^*} + \frac{(x_1^* - x_2^*)}{x_2^*} \right)$$

$$(S_1 + S_2) - (x_1^* + x_2^*) = \frac{d}{\rho} \left(\frac{(x_2^* - x_1^*)x_2^* + (x_1^* - x_2^*)x_1^*}{x_1^*x_2^*} \right)$$

$$(S_1 + S_2) - (x_1^* + x_2^*) = \frac{d}{\rho} \left(\frac{(x_2^* - x_1^*)^2}{x_1^*x_2^*} \right)$$

$$\frac{dx_1}{dt} = \rho x_1(S_1 - x_1) + d(x_2 - x_1)$$

$$\frac{dx_2}{dt} = \rho x_2(S_2 - x_2) + d(x_1 - x_2)$$

$$\frac{dx_1}{dt} = \rho x_1(S_1 - x_1) + d(\sigma_1(x_1)x_2 - x_1)$$

$$\frac{dx_2}{dt} = \rho x_2(S_2 - x_2) + d(\sigma_2(x_2)x_1 - x_2)$$

$$\sigma_i(x_i) = 1 \text{ si } x_i < S_i \text{ 0 sinon}$$

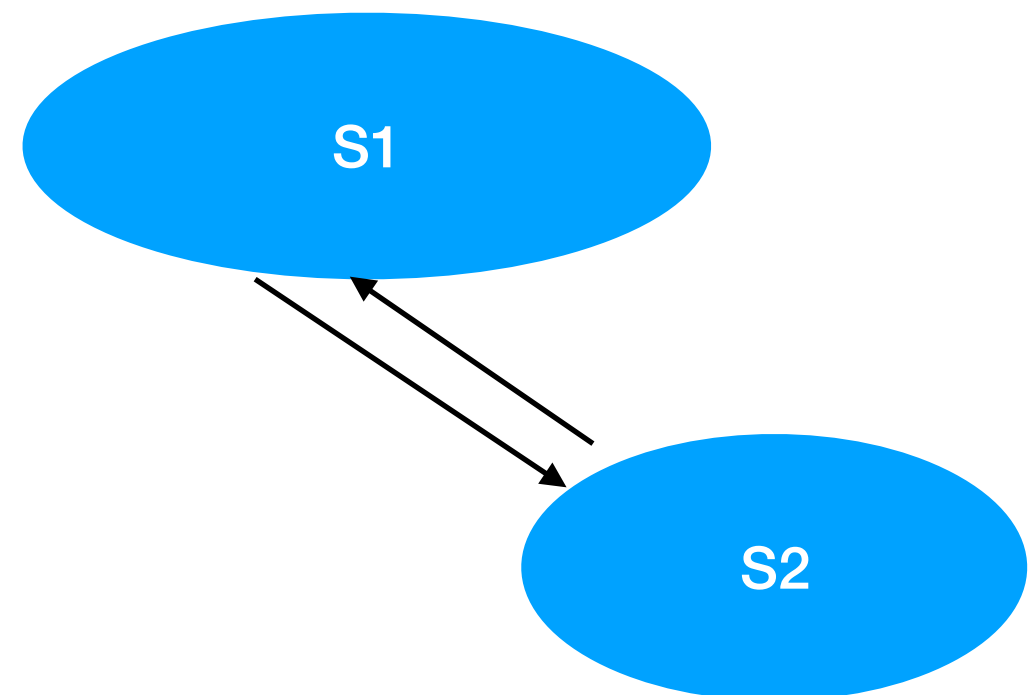
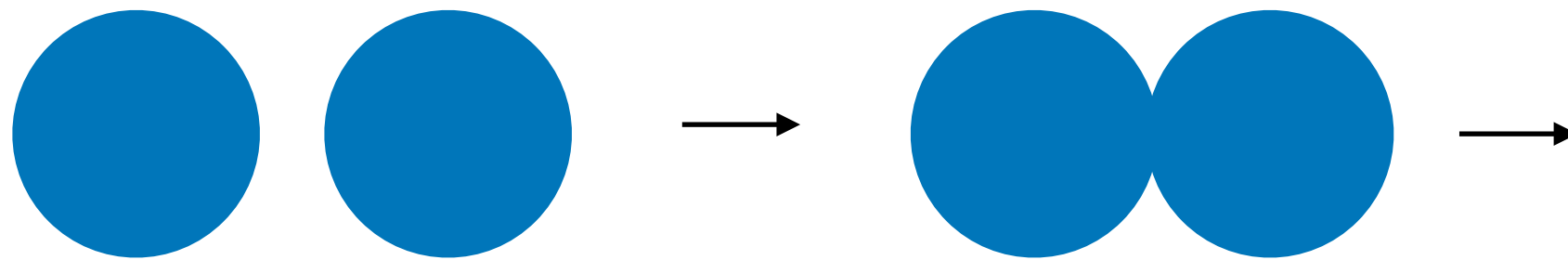
Erreur de modélisation

Mais valable pour l'interprétation

$$\frac{dx}{dt} = \rho x - \alpha x^2$$

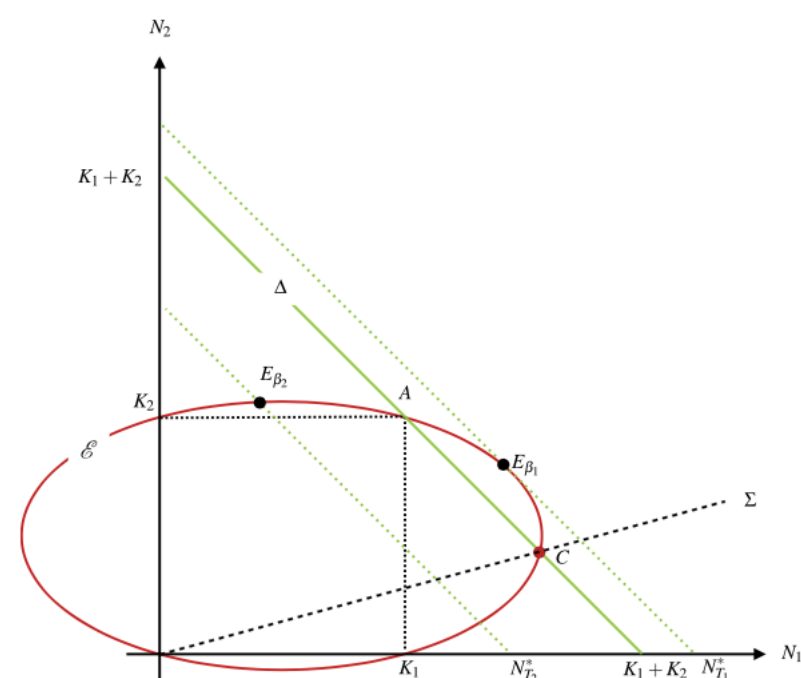
Mais valable pour l'interprétation
Compétition intra-spécifique

$$\frac{dx}{dt} = \rho x - \alpha x^2$$



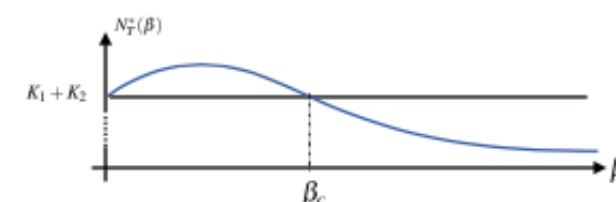
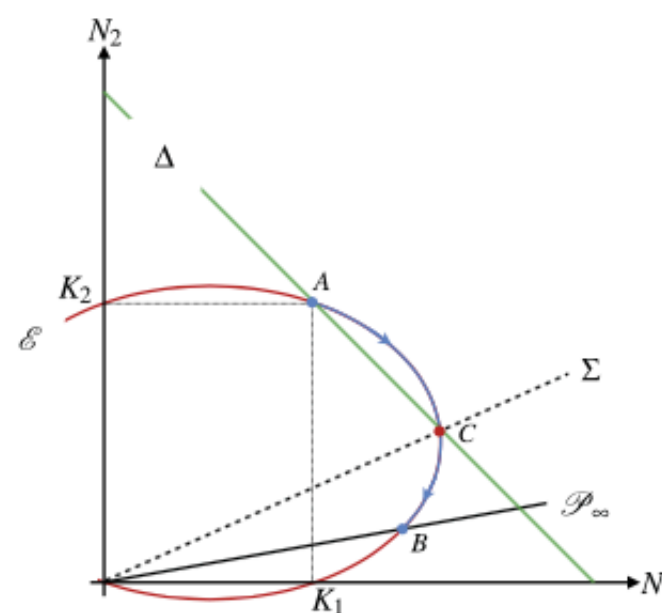
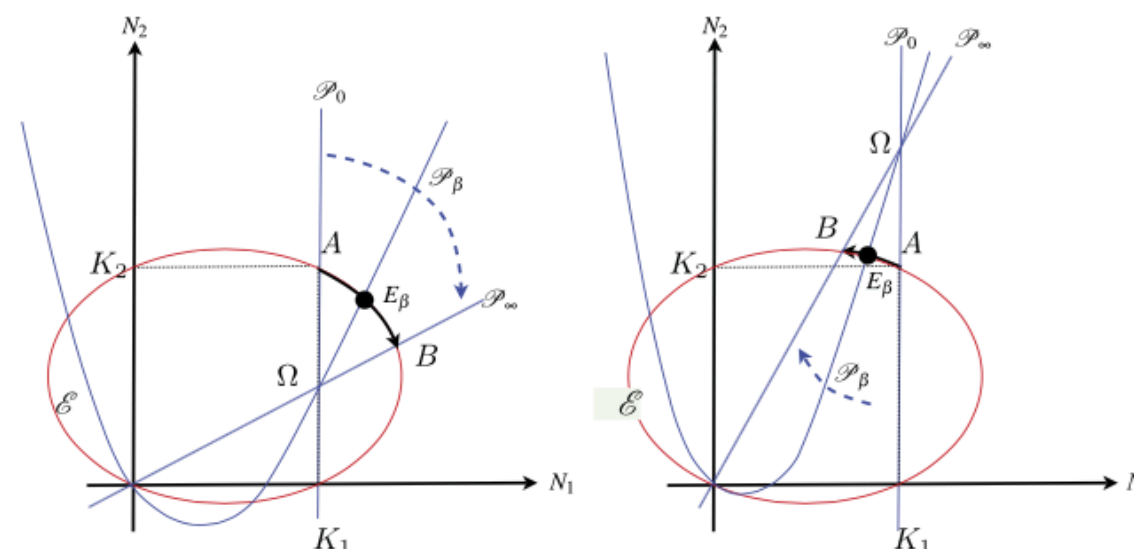
Is dispersal always beneficial to carrying capacity? New insights from the multi-patch logistic equation

Roger Arditi^{a,d,*}, Claude Lobry^{b,e}, Tewfik Sari^{c,f}



Asymmetric dispersal in the multi-patch logistic equation

Roger Arditi^{a, b}, Claude Lobry^c, Tewfik Sari^{d, e}



$$\frac{dx_1}{dt} = r_1 x_1 \left(+1 - \frac{x_1}{K_1} \right) + m(x_2 - x_1)$$

$$\frac{dx_2}{dt} = r_2 x_2 \left(-1 - \frac{x_2}{K_2} \right) + m(x_1 - x_2)$$

Equilibres

$$0 = r_1 x_1 \left(+1 - \frac{x_1}{K_1} \right) + m(x_2 - x_1)$$

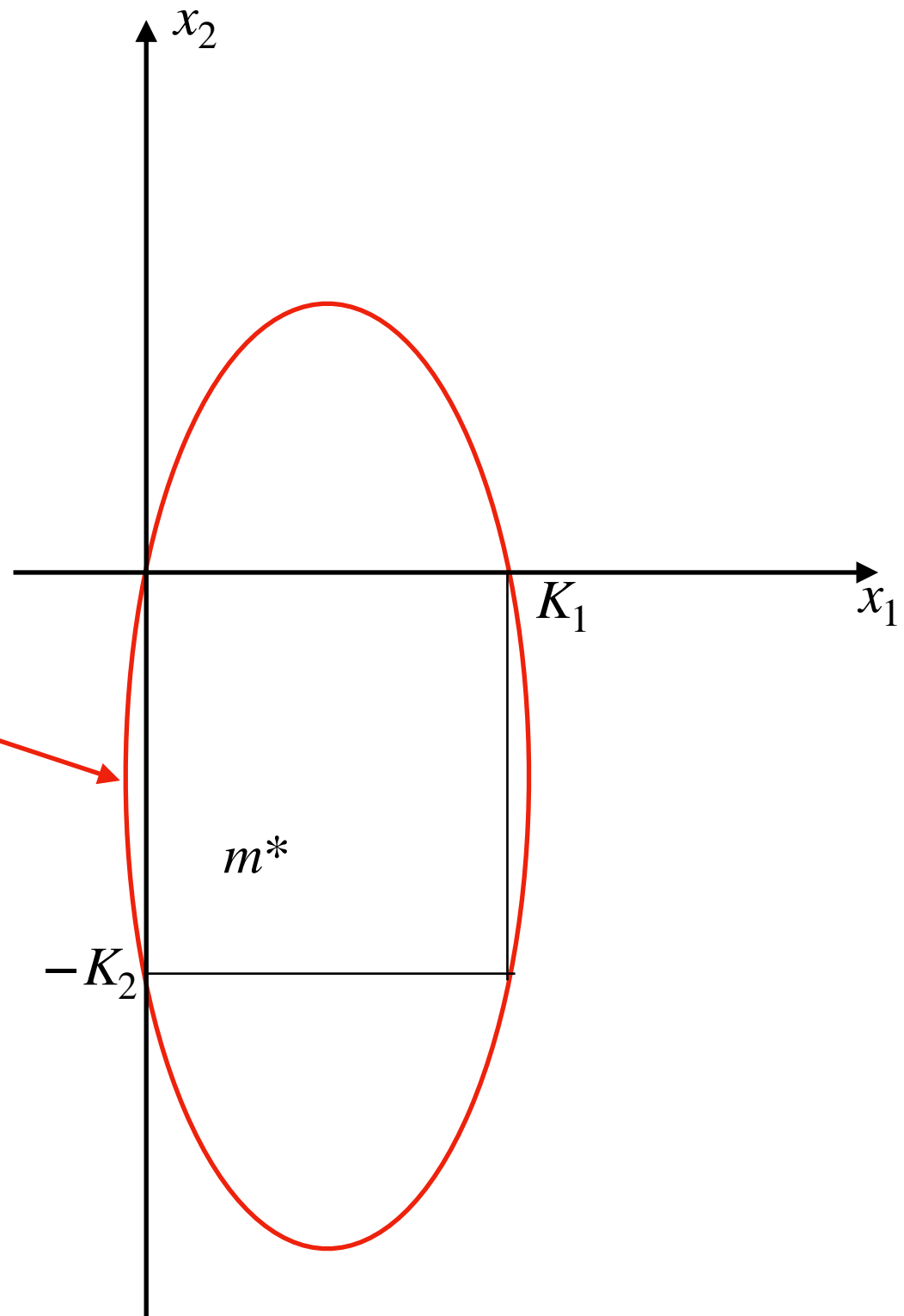
$$0 = r_2 x_2 \left(-1 - \frac{x_2}{K_2} \right) + m(x_1 - x_2)$$

Isoclines

$$0 = r_1 x_1 \left(+1 - \frac{x_1}{K_1} \right) + m(x_2 - x_1)$$

$$+ \quad 0 = r_2 x_2 \left(-1 - \frac{x_2}{K_2} \right) + m(x_1 - x_2)$$

$$0 = r_1 x_1 - r_2 x_2 - r_1 \frac{x_1^2}{K_1} - r_2 \frac{x_2^2}{K_2}$$

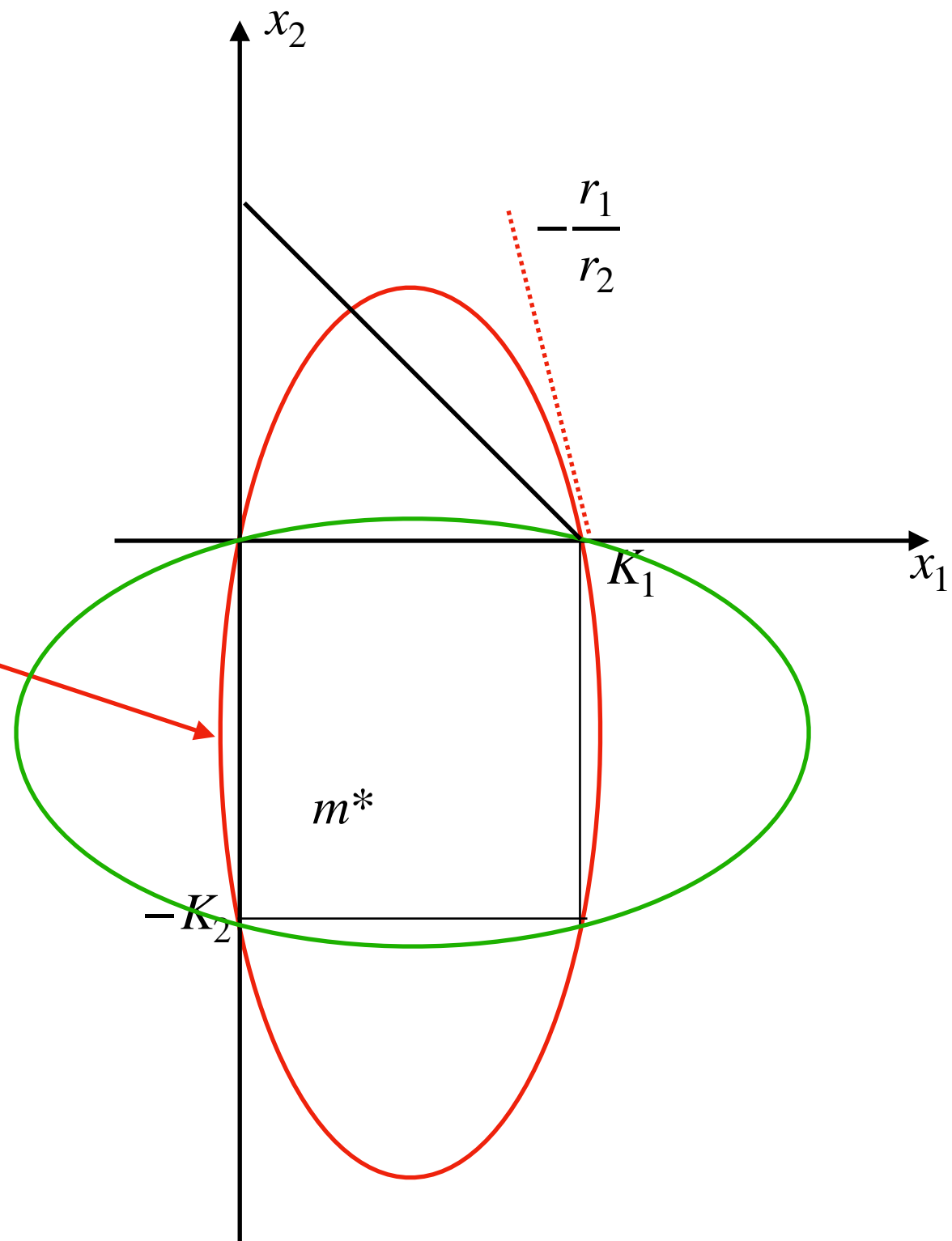


$$0 = r_1 x_1 \left(+1 - \frac{x_1}{K_1} \right) + m(x_2 - x_1)$$

$$0 = r_2 x_2 \left(-1 - \frac{x_2}{K_2} \right) + m(x_1 - x_2)$$

+

$$0 = r_1 x_1 - r_2 x_2 - r_1 \frac{x_1^2}{K_1} - r_2 \frac{x_2^2}{K_2}$$



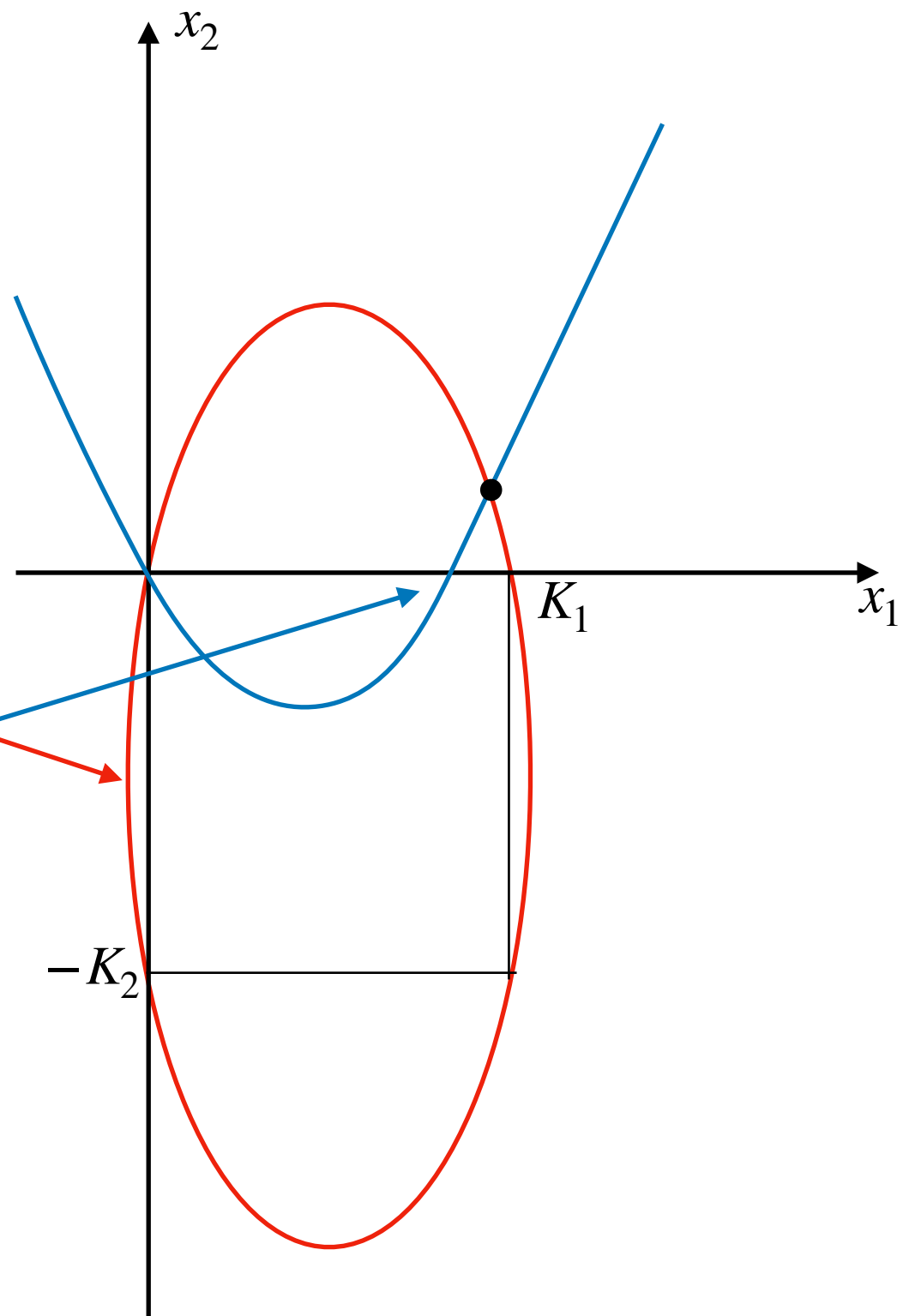
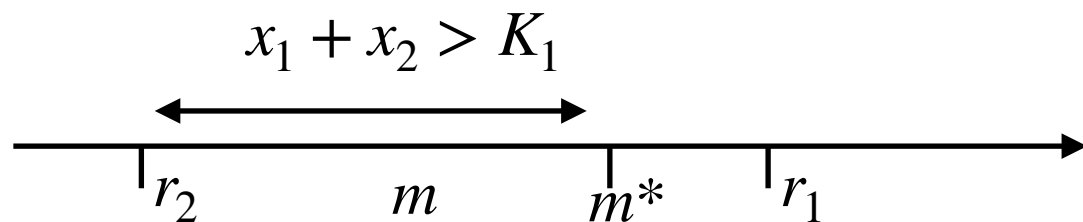
$$0 = r_1 x_1 \left(+1 - \frac{x_1}{K_1} \right) + m(x_2 - x_1)$$

$$+ \quad 0 = r_2 x_2 \left(-1 - \frac{x_2}{K_2} \right) + m(x_1 - x_2)$$

$$0 = r_1 x_1 - r_2 x_2 - r_1 \frac{x_1^2}{K_1} - r_2 \frac{x_2^2}{K_2}$$

$$x_2 = \left(1 - \frac{r_1}{m} \right) x_1 + \frac{r_1}{m K_1} x_1^2$$

$$x_2 = 0 \Rightarrow x_1 = K_1 \left(1 - \frac{m}{r_1} \right)$$



$$0 = r_1 x_1 \left(+1 - \frac{x_1}{K_1} \right) + m(x_2 - x_1)$$

$$0 = r_2 x_2 \left(-1 - \frac{x_2}{K_2} \right) + m(x_1 - x_2)$$

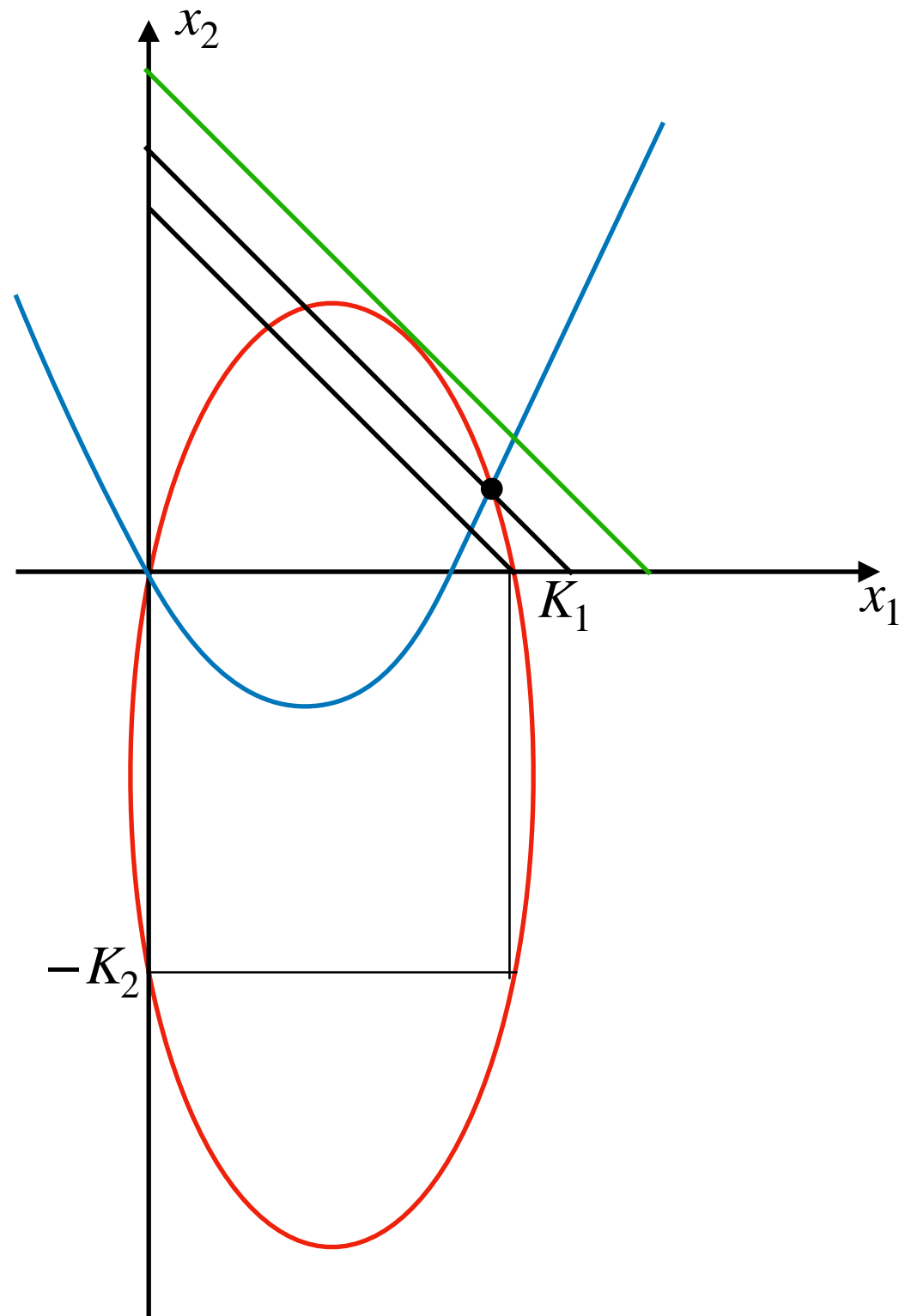
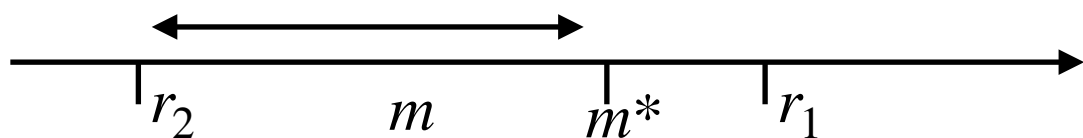
+

$$0 = r_1 x_1 - r_2 x_2 - r_1 \frac{x_1^2}{K_1} - r_2 \frac{x_2^2}{K_2}$$

$$x_2 = \left(1 - \frac{r_1}{m} \right) x_1 + \frac{r_1}{m K_1} x_1^2$$

$$x_2 = 0 \Rightarrow x_1 = K_1 \left(1 - \frac{m}{r_1} \right)$$

$$x_1 + x_2 > K_1$$



$$0 = r_1 x_1 \left(+1 - \frac{x_1}{K_1} \right) + m(x_2 - x_1)$$

$$0 = r_2 x_2 \left(-1 - \frac{x_2}{K_2} \right) + m(x_1 - x_2)$$

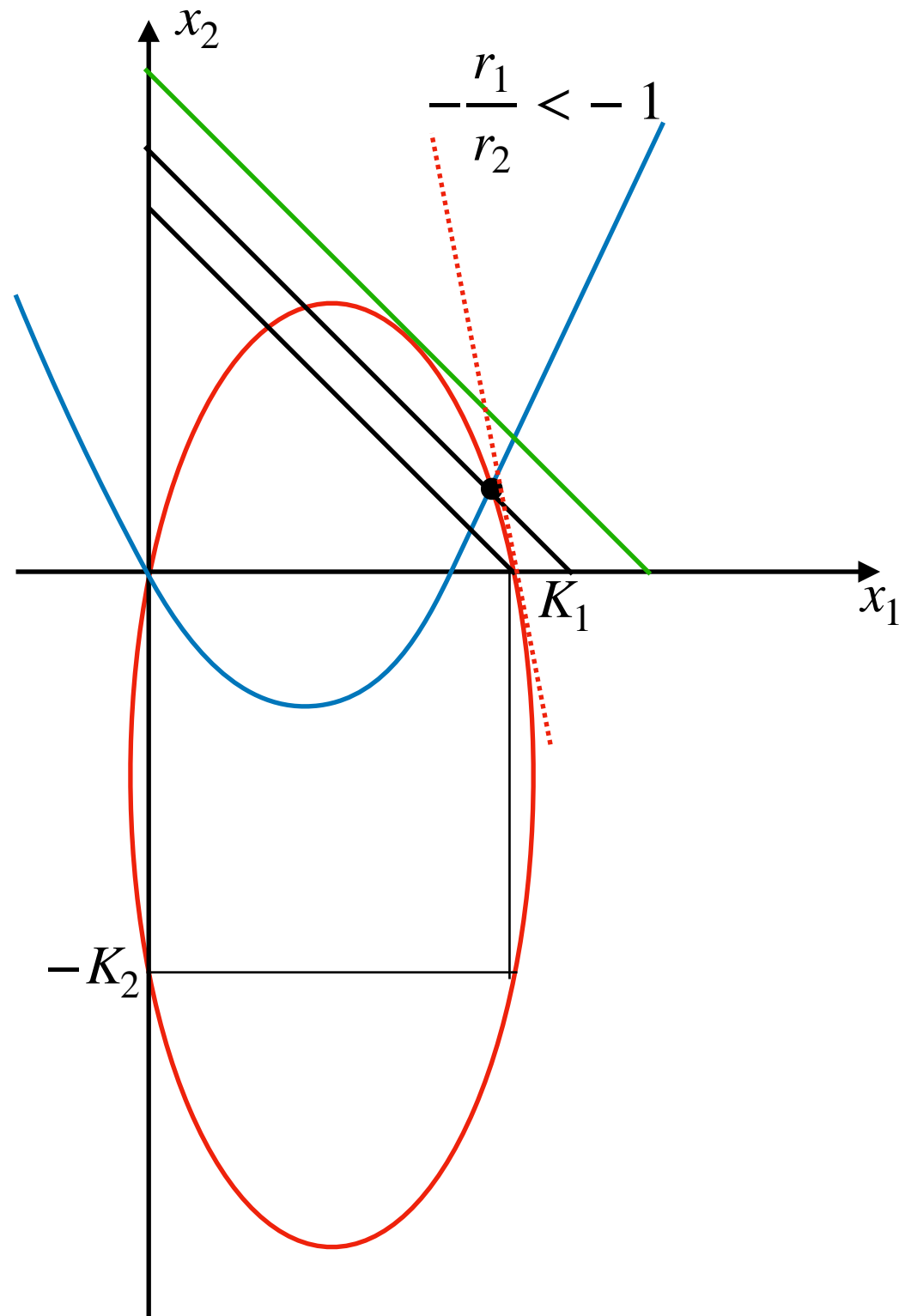
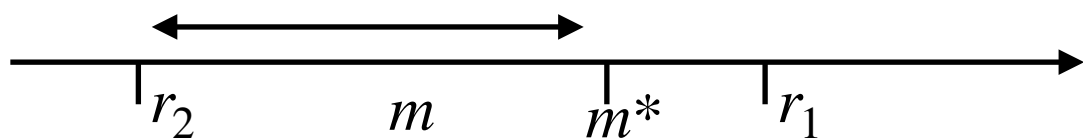
+

$$0 = r_1 x_1 - r_2 x_2 - r_1 \frac{x_1^2}{K_1} - r_2 \frac{x_2^2}{K_2}$$

$$x_2 = \left(1 - \frac{r_1}{m} \right) x_1 + \frac{r_1}{m K_1} x_1^2$$

$$x_2 = 0 \Rightarrow x_1 = K_1 \left(1 - \frac{m}{r_1} \right)$$

$$x_1 + x_2 > K_1$$



$$0 = r_1 x_1 \left(+1 - \frac{x_1}{K_1} \right) + m(x_2 - x_1)$$

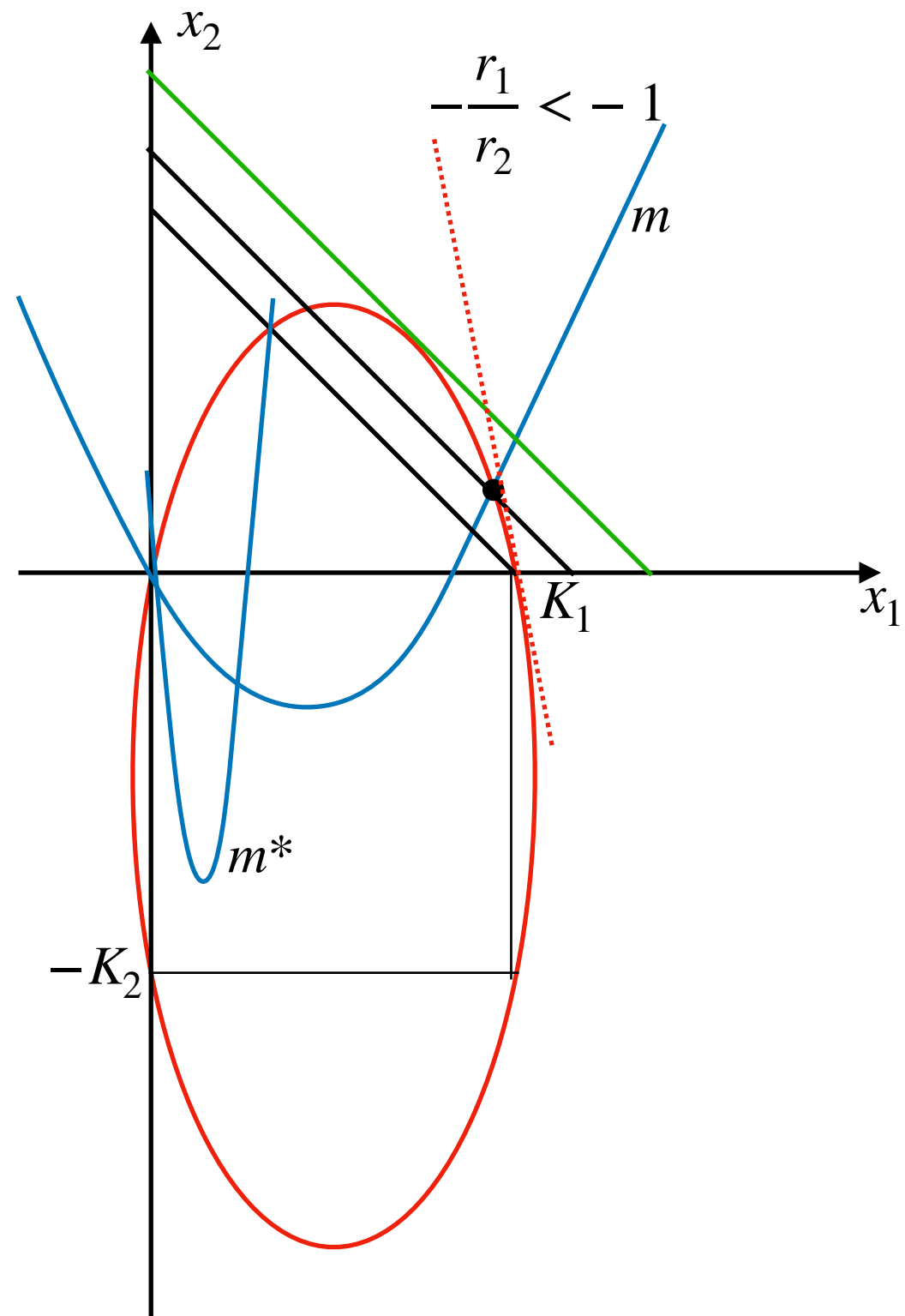
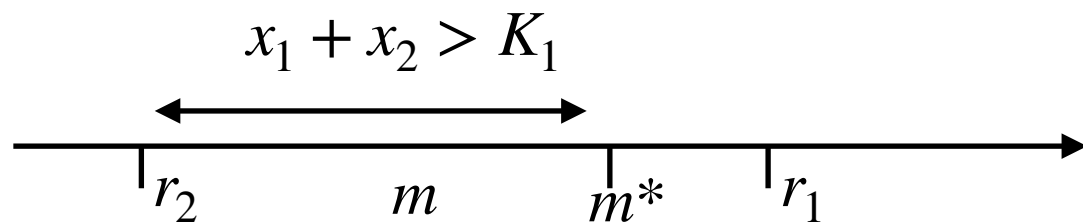
$$0 = r_2 x_2 \left(-1 - \frac{x_2}{K_2} \right) + m(x_1 - x_2)$$

+

$$0 = r_1 x_1 - r_2 x_2 - r_1 \frac{x_1^2}{K_1} - r_2 \frac{x_2^2}{K_2}$$

$$x_2 = \left(1 - \frac{r_1}{m} \right) x_1 + \frac{r_1}{m K_1} x_1^2$$

$$x_2 = 0 \Rightarrow x_1 = K_1 \left(1 - \frac{m}{r_1} \right)$$



Plus de deux sites

Manuscript submitted to
AIMS' Journals
Volume **X**, Number **0X**, XX **200X**

[doi:10.3934/xx.xx.xx.xx](https://doi.org/10.3934/xx.xx.xx.xx)

pp. **X–XX**



Revista Integración
Escuela de Matemáticas
Universidad Industrial de Santander
Vol. 40, N° 1, 2022, pág. 25–57



THE MULTI-PATCH LOGISTIC EQUATION

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(Communicated by Frithjof Lutscher)

The multi-patch logistic equation with asymmetric migration

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Effect of dispersal in single-species discrete diffusion systems with source-sink populations

Bilel Elbetch

$$\frac{dx_1}{dt} = \rho x_1(S_1 - x_1) + d(x_2 - x_1)$$

$$\frac{dx_2}{dt} = \rho x_2(S_2 - x_2) + d(x_1 - x_2)$$

$$\begin{pmatrix} \rho S_1 - d & d \\ d & \rho S_2 - d \end{pmatrix} = \begin{pmatrix} a_1 & d \\ d & a_2 \end{pmatrix}$$

$$\lambda^2 - (a_1 + a_2)\lambda + a_1 a_2 - d^2 = 0 \implies \Delta = (a_1 + a_2)^2 - 4a_1 a_2 + 4d^2 = (a_1 - a_2)^2 + 4d^2$$

Deux racines réelles, une “racine dominante”

$$\begin{aligned}\frac{dx_1}{dt} &= \rho x_1(S_1 - x_1) + d(x_2 - x_1) \\ \frac{dx_2}{dt} &= \rho x_2(S_2 - x_2) + d(x_1 - x_2)\end{aligned}$$

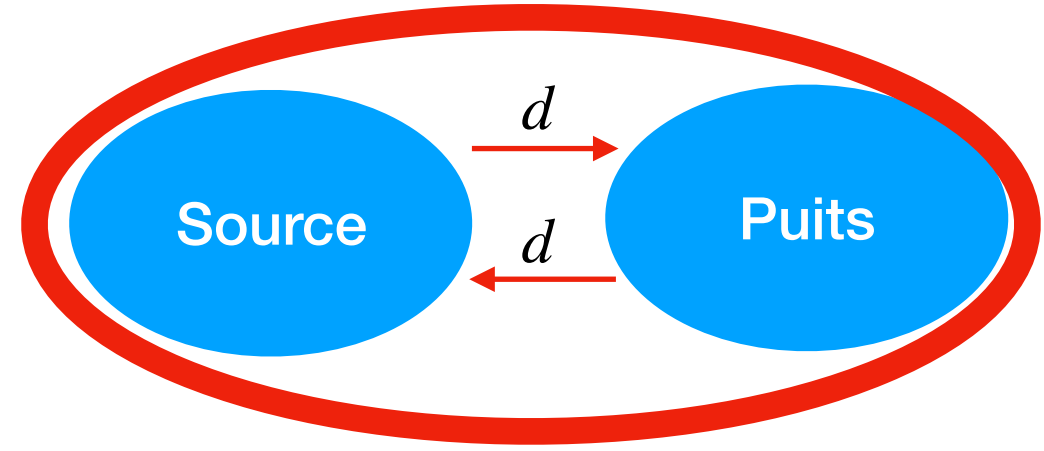
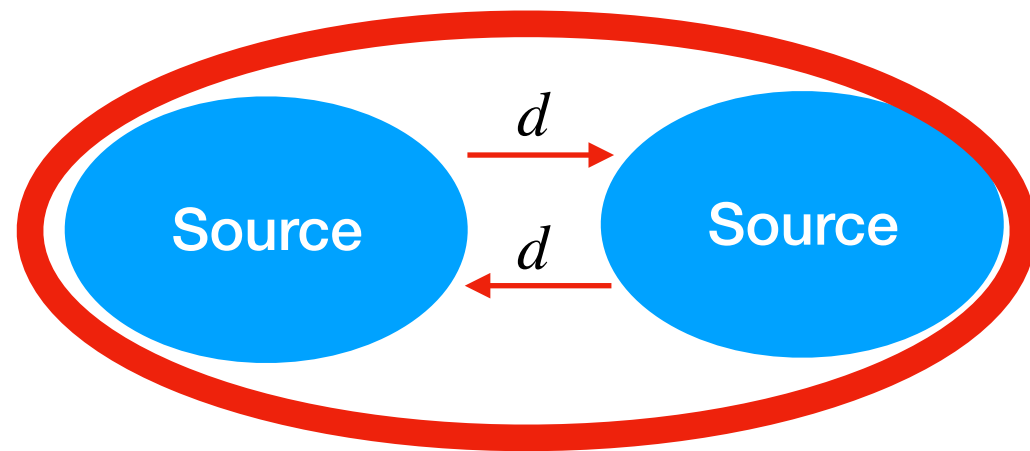
$$\begin{pmatrix} \rho S_1 - d & d \\ d & \rho S_2 - d \end{pmatrix} = \begin{pmatrix} a_1 & d \\ d & a_2 \end{pmatrix}$$

**Algèbre linéaire théorie de
Perron Frobenius
1907 1912**

$$\lambda^2 - (a_1 + a_2)\lambda + a_1 a_2 - d^2 = 0 \implies \Delta = (a_1 + a_2)^2 - 4a_1 a_2 + 4d^2 = (a_1 - a_2)^2 + 4d^2$$

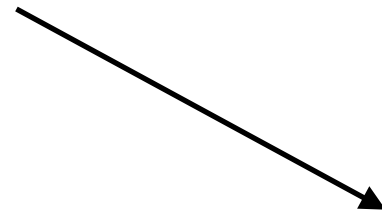
Deux racines réelles, une "racine dominante"

$$\frac{dX}{dt} = AX \quad X \in \mathbb{R}^n \quad i \neq j \Rightarrow a_{ij} \geq 0 \quad \text{Primitive réursive}$$



$$(x_1^*, x_2^*) \rightarrow x_1^* + x_2^*$$

~~Carrying capacity~~



Total Equilibrium Population (TEP)

Parenthèse : mathématiques ≠ dynamique des populations

$$\frac{dx}{dt} = rx(S - x)$$

$$x = Sy \implies \frac{dy}{dt} = rSx(1 - x)$$

$$z(t) = y(t/rS) \implies \frac{dz}{dt} = z(1 - z)$$

$$\frac{dx}{dt} = rx(S - x)$$

$$\frac{dx}{dt} = rx \left(1 - \frac{x}{K} \right)$$

Carrying capacity

Taux de croissance intrinsèque

The **carrying capacity** of an **environment** is the maximum population size of a biological **species** that can be sustained by that specific environment, given the food, **habitat**, **water**, and other **resources** available.

Parenthèse : mathématiques ≠ dynamique des populations

$$\frac{dx}{dt} = rx(S - x)$$

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$$\frac{dx}{dt} = rx(S - x)$$

$$\frac{dx}{dt} = rx \left(1 - \frac{x}{K} \right)$$

Carrying capacity

Taux de croissance intrinsèque

$$\frac{dx}{dt} = \rho x - \alpha x^2$$

Compétition intraspécifique

Taux de croissance intrinsèque

The **carrying capacity** of an **environment** is the maximum population size of a biological **species** that can be sustained by that specific environment, given the food, **habitat**, **water**, and other **resources** available.

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Ecology, 76(2), 1995, pp. 336–343
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LOGISTIC THEORY OF FOOD WEB DYNAMICS¹

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Compétition sur deux sites

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Coupling in Predator–Prey Dynamics: Ratio-Dependence

ROGER ARDITI[†] AND LEV R. GINZBURG[‡]

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Modèle ratio dépendant

From Lotka–Volterra to Arditi–Ginzburg: 90 Years of Evolving Trophic Functions

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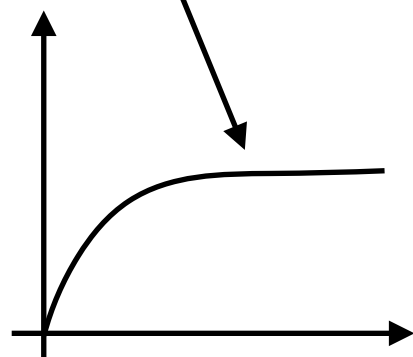
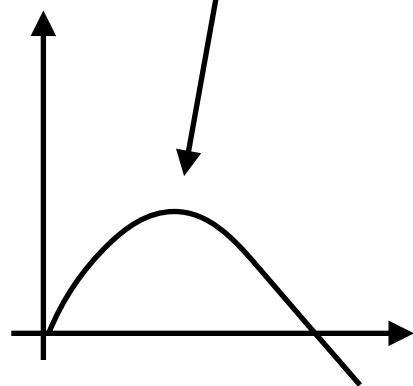
^{**}*e-mail: ltitova@sfedu.ru*

Received March 18, 2018

$$\frac{dx}{dt} = f(x) - \mu(x)y$$

$$\frac{dy}{dt} = (\mu(x) - m)y$$

Rosenzweig-MacArthur



$$\frac{dx}{dt} = f(x) - \mu(x/y)y$$

$$\frac{dy}{dt} = (\mu(x/y) - m)y$$

Modèle *ratio dépendant*

**Consommation de blé en Inde = 100 MT
en France = 15 MT**

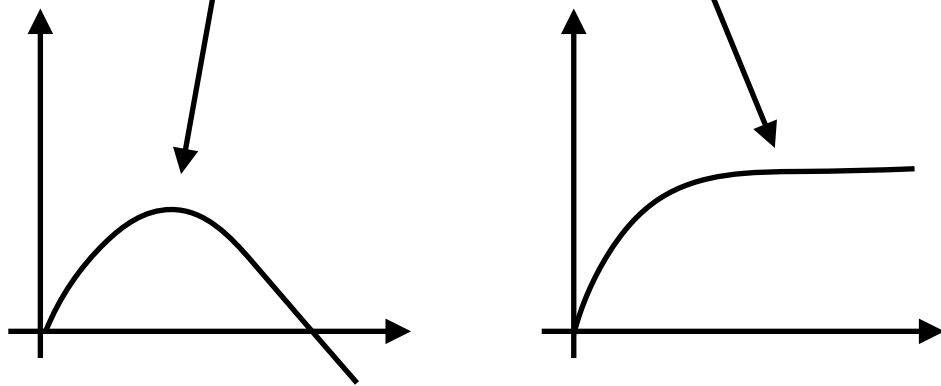
$$\frac{dx}{dt} = D(x_{in} - x) - \mu(x)y$$

$$\frac{dy}{dt} = (\mu(x) - D)y$$

$$\frac{dx}{dt} = f(x) - \mu(x)y$$

$$\frac{dy}{dt} = (\mu(x) - m)y$$

Rosenzweig-MacArthur



$$\frac{dx}{dt} = f(x) - \mu(x/y)y$$

$$\frac{dy}{dt} = (\mu(x/y) - m)y$$

Modèle *ratio dépendant*

About Deterministic Extinction in Ratio-dependent Predator-Prey Models

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Arino 1947-2003

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Bulletin of Mathematical Biology (1999) **61**, 19–32

Article No. bulm.1998.0072

Available online at <http://www.idealibrary.com> on IDEAL

Following Arino's death, several tributes to him have been made. For example on 9-10 January 2004, the Cadi Ayyad University and the International Centre for Dynamical Systems in Marrakech organized two days of mathematics in his memory. In May 2004, the SFBT awarded the Prix Ovide Arino during its first international conference. In June 2004, the Université Abou Bekr Belkaid, Tlemcen, Algeria, awarded the Prix Ovide Arino 2004 to Tarik Touaoula.

Arino 1947-2003

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Compétition sur deux sites

**Migrations in the Rosenzweig-MacArthur
model and the “atto-fox” problem**

ARIMA Journal, vol. 20, pp. 95-125 (2015)

Claude Lobry* — Tewfik Sari **

**Pourquoi Rosenzweig MacArthur
et pas ratio dépendant ?**

Pourquoi ARIMA ?

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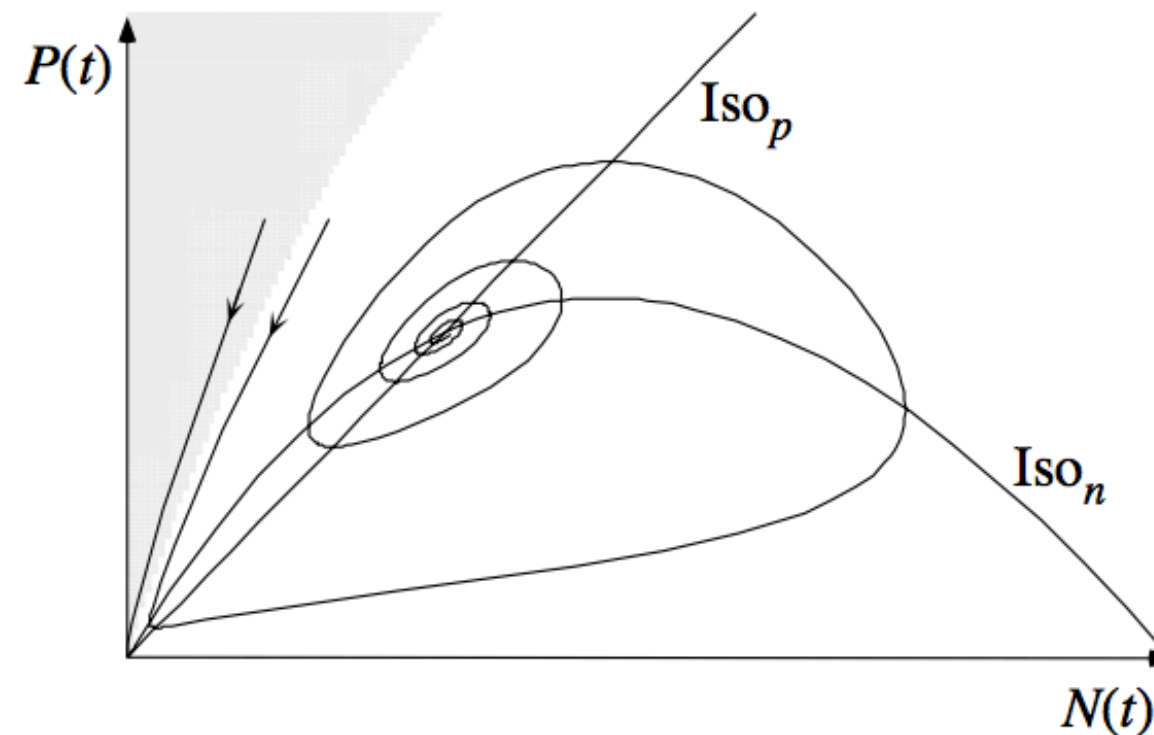
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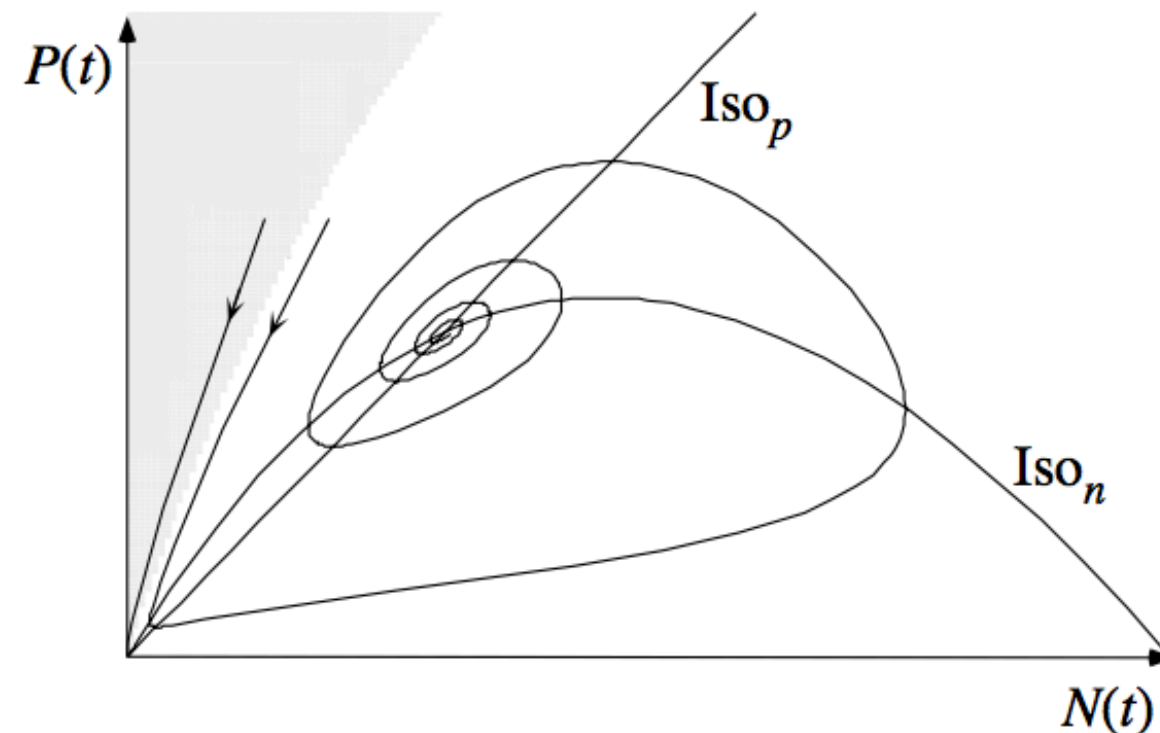
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On a le droit d'explorer mathématiquement un modèle même si on le sait peu pertinent.

Les propriétés dégagées peuvent renforcer ou modifier l'idée de la pertinence

Compétition sur deux sites

On a le droit d'explorer mathématiquement un modèle même si on le sait peu pertinent.

Faire les mathématiques qui nous plaisent est un droit imprescriptible.

Déclaration universelle des droits de l'homme



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2. Chacun a droit à la protection des intérêts moraux et matériels découlant de toute production scientifique, littéraire ou artistique dont il est l'auteur.
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**Les propriétés dégagées peuvent renforcer
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La revue est née d'une collaboration scientifique Nord/Sud menée depuis plus de vingt ans dans le sillage des activités **CARI (Colloque Africain pour la Recherche en Informatique et mathématiques appliquées)**. Elle permet de donner un coup de projecteur sur les résultats de recherche issus de ces coopérations. Le domaine scientifique recouvre tous les sujets de recherche de l'informatique et des mathématiques appliquées.

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Hammamet, Tunisie



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16^{ème} Colloque Africain sur la Recherche en Informatique et Mathématiques Appliquées

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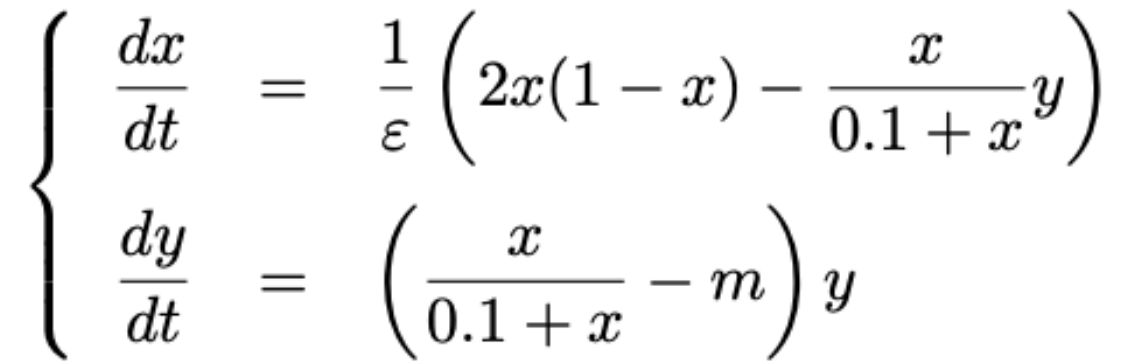


B. Philippe



M. Tchunte





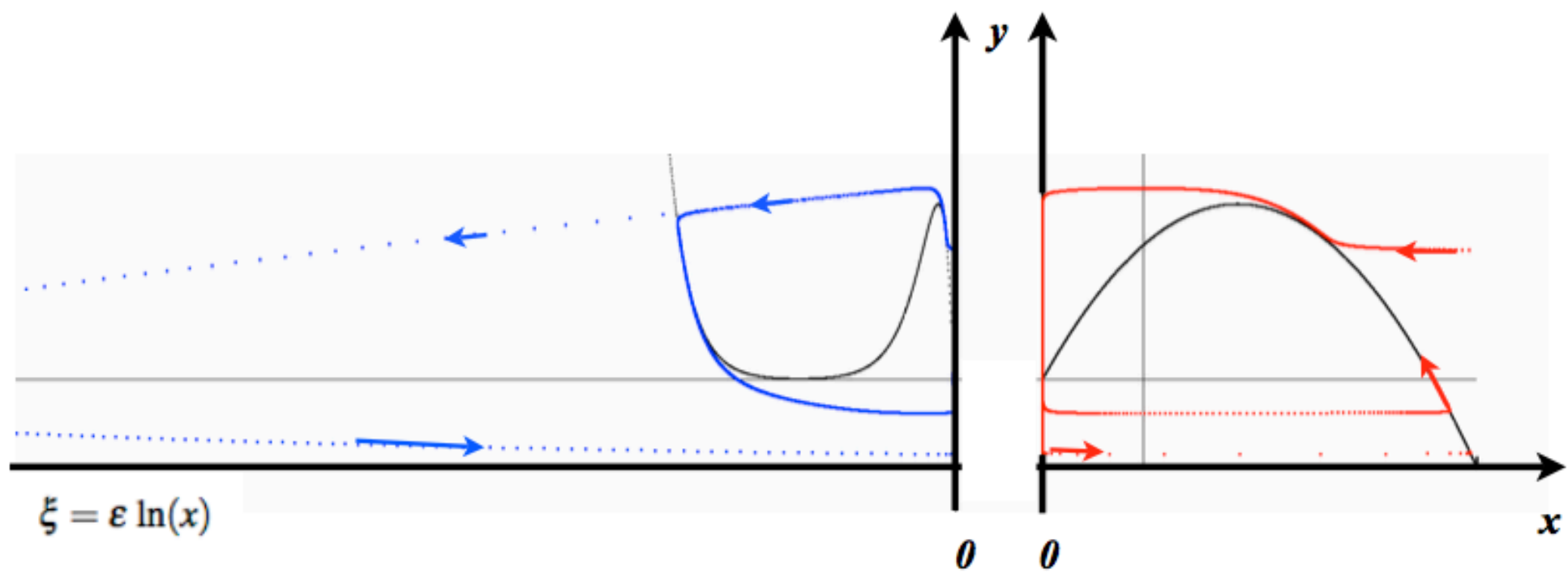
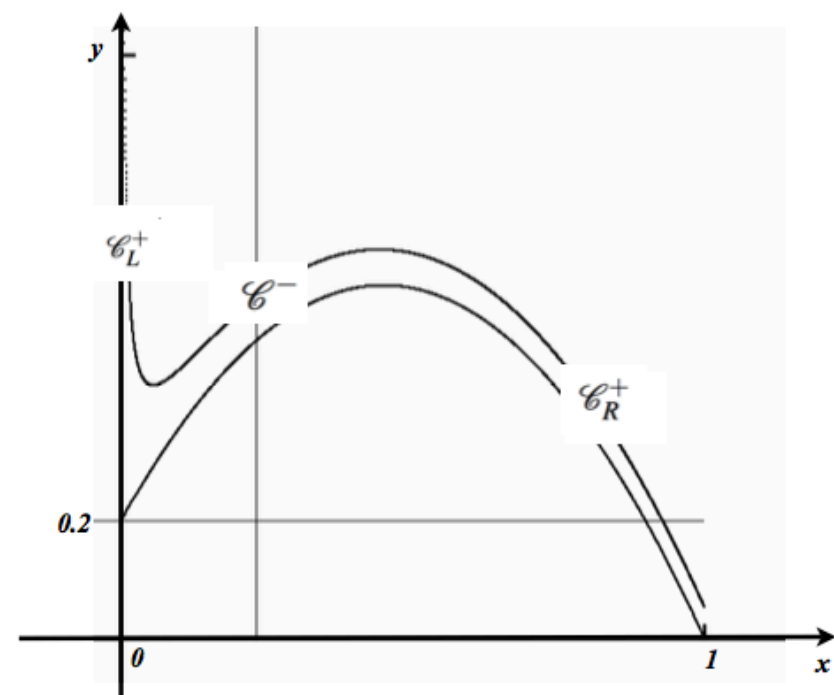
$$\xi = \varepsilon \ln(x) \qquad x = \exp\left(\frac{\xi}{\varepsilon}\right)$$

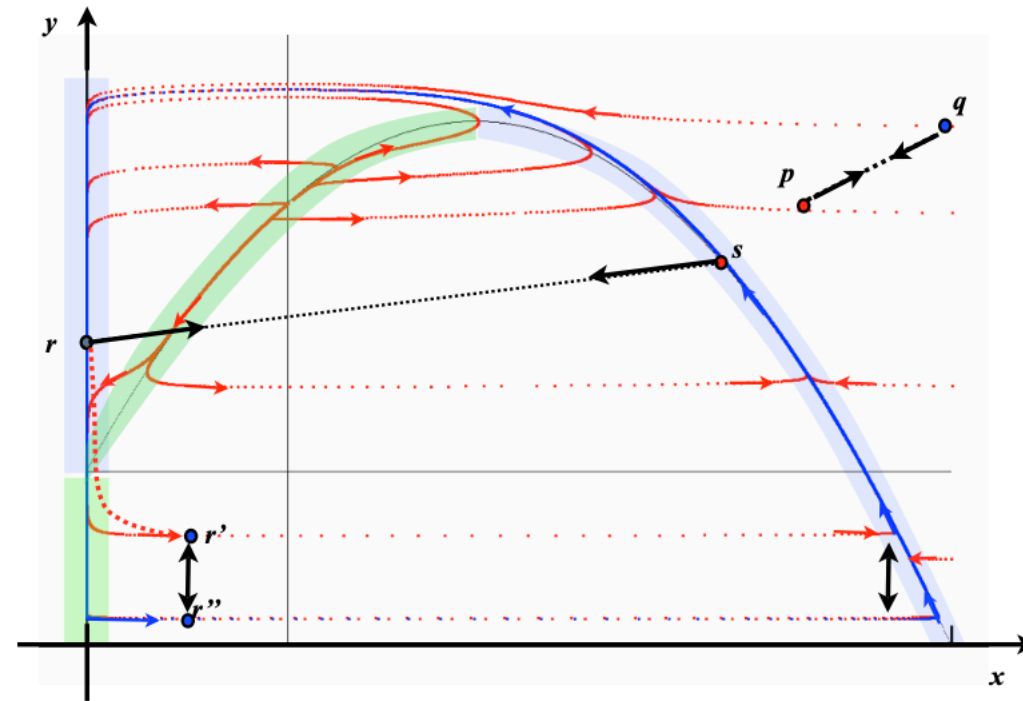
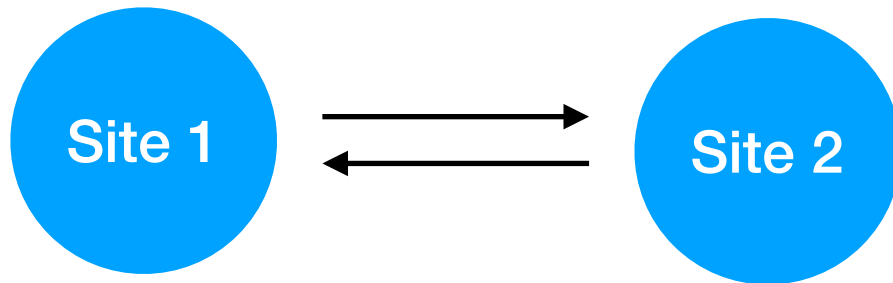
$$\begin{cases} \frac{d\xi}{dt} = 2 \left(1 - \exp \left(\frac{\xi}{\varepsilon} \right) \right) - \frac{y}{0.1 + \exp \left(\frac{\xi}{\varepsilon} \right)} \\ \frac{dy}{dt} = \left(\frac{\exp \left(\frac{\xi}{\varepsilon} \right)}{0.1 + \exp \left(\frac{\xi}{\varepsilon} \right)} - m \right) y \end{cases} \approx \begin{cases} \frac{d\hat{\xi}}{dt} = 2 - 10 \hat{y} \\ \frac{d\hat{y}}{dt} = -m \hat{y} \end{cases}$$

y_0	0.4	0.5	0.6	0.7
$\exp(\frac{\xi_{min}}{\varepsilon})$	1.66 10^{-9}	1.79 10^{-16}	2.14 10^{-24}	5.34 10^{-33}
x_{min}	1.62 10^{-9}	1.69 10^{-16}	1.90 10^{-24}	4.31 10^{-33}

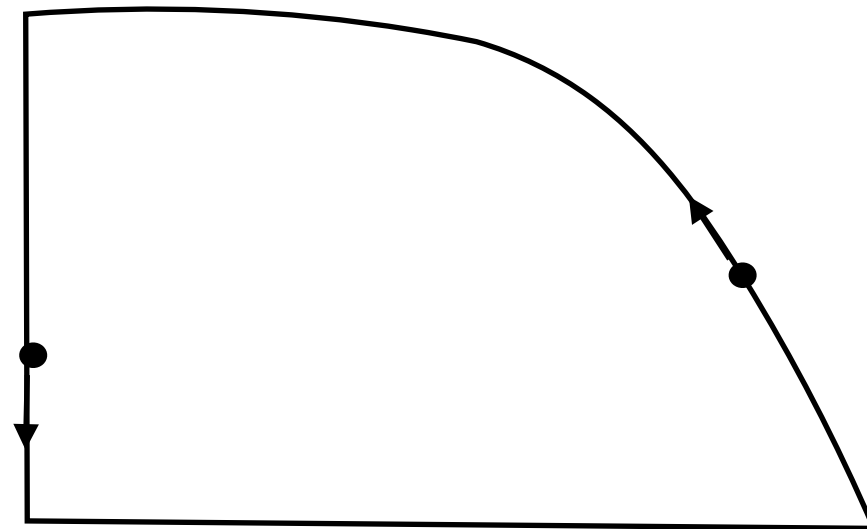
Values of the parameters : $\varepsilon = 0.05$; $m = 0.7$; $x_0 = 0.05$; $\xi_0 = \varepsilon \log(x_0)$

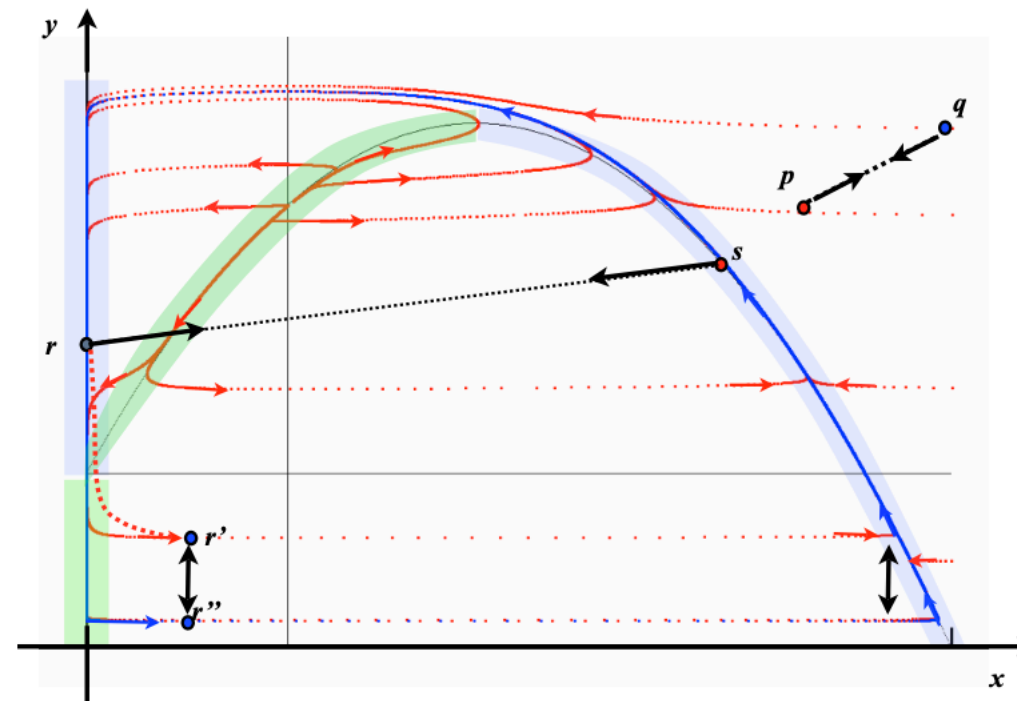
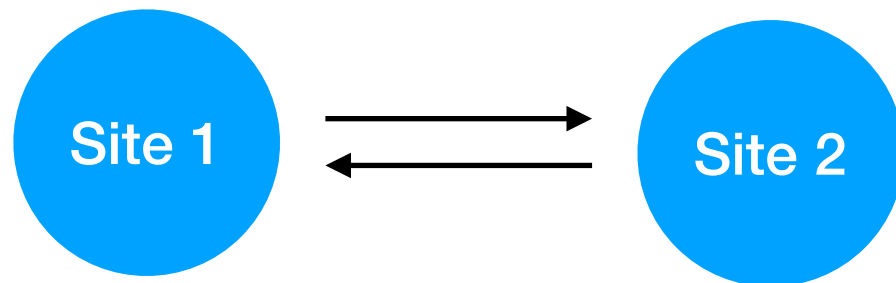
$$\begin{cases} \frac{dx}{dt} = \frac{1}{\varepsilon} \left(2x(1-x) - \frac{x}{0.1+x}y \right) + I \\ \frac{dy}{dt} = \left(\frac{x}{0.1+x} - m \right) y \end{cases}$$



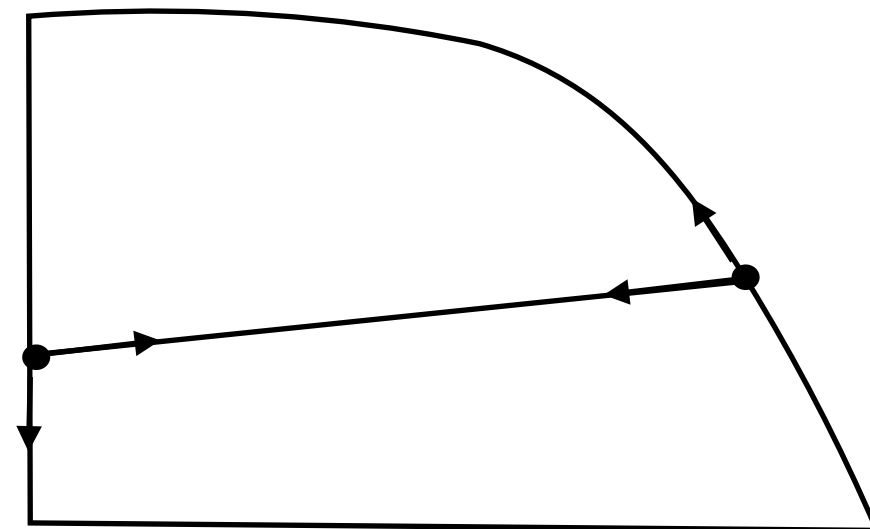


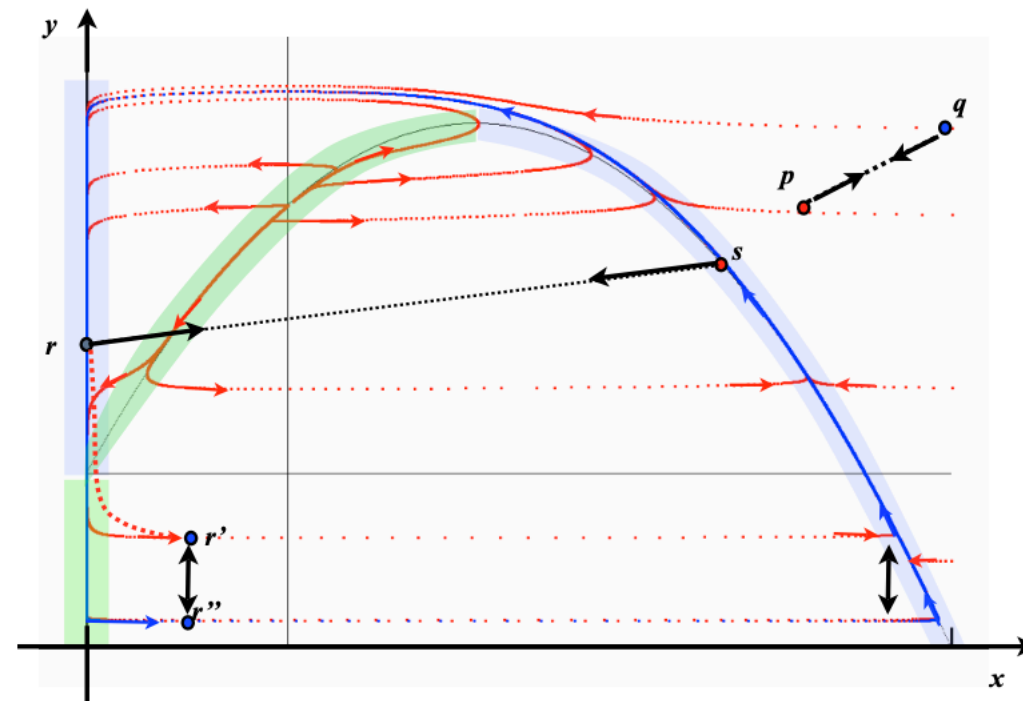
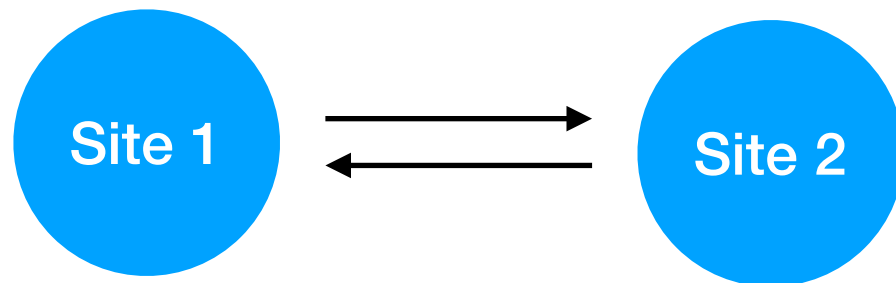
$$\left\{ \begin{array}{l} \frac{dx_1}{dt} = \frac{1}{\varepsilon}(x_1 g(x_1) - h(x_1)y_1) + k(x_2 - x_1) \\ \frac{dy_1}{dt} = (h(x_1) - m)y_1 + k(y_2 - y_1) \\ \frac{dx_2}{dt} = \frac{1}{\varepsilon}(x_2 g(x_2) - h(x_2)y_2) + k(x_1 - x_2) \\ \frac{dy_2}{dt} = (h(x_2) - m)y_2 + k(y_1 - y_2) \end{array} \right.$$



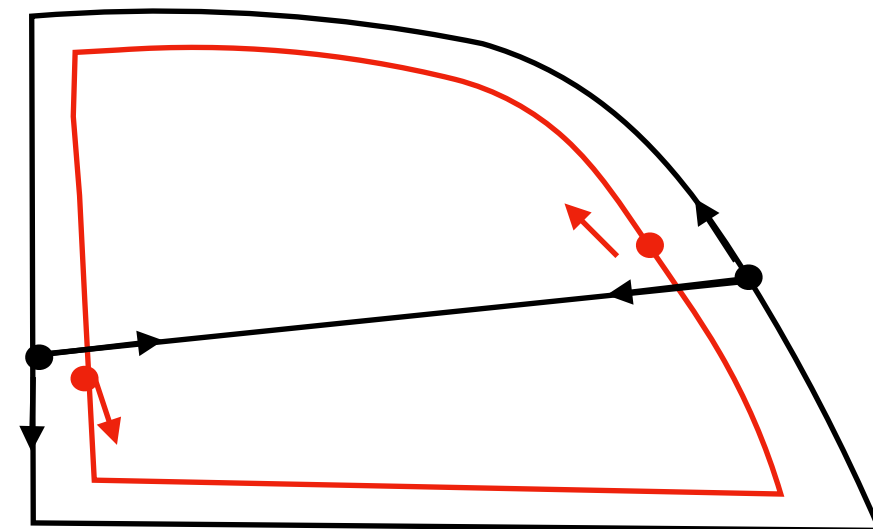


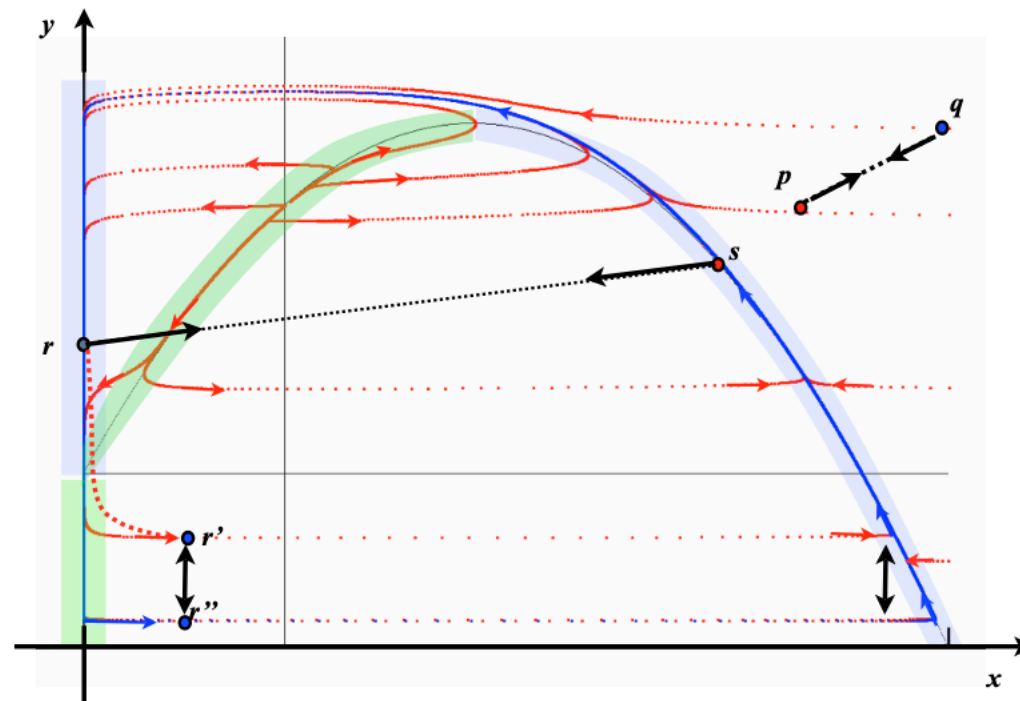
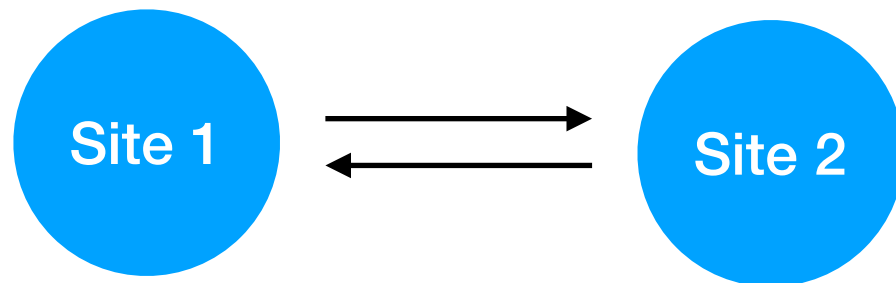
$$\left\{ \begin{array}{l} \frac{dx_1}{dt} = \frac{1}{\varepsilon} (x_1 g(x_1) - h(x_1) y_1) + k(x_2 - x_1) \\ \frac{dy_1}{dt} = (h(x_1) - m) y_1 + k(y_2 - y_1) \\ \frac{dx_2}{dt} = \frac{1}{\varepsilon} (x_2 g(x_2) - h(x_2) y_2) + k(x_1 - x_2) \\ \frac{dy_2}{dt} = (h(x_2) - m) y_2 + k(y_1 - y_2) \end{array} \right.$$





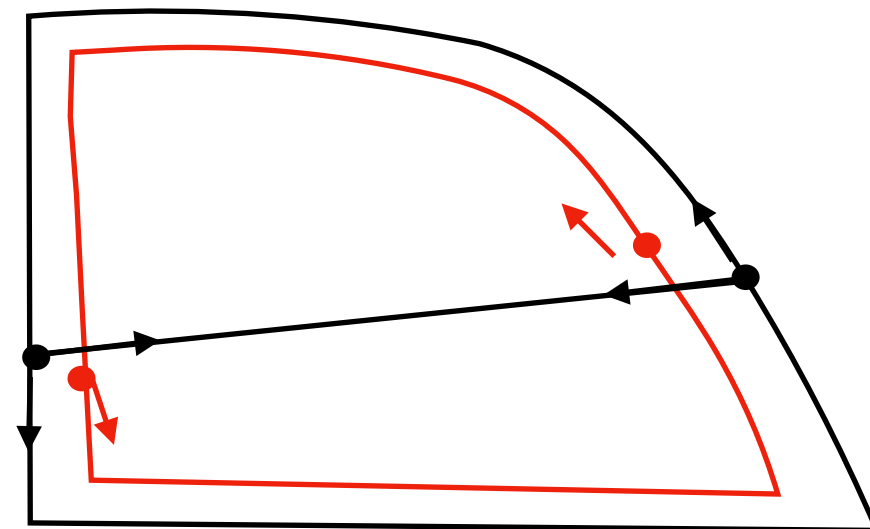
$$\left\{ \begin{array}{l} \frac{dx_1}{dt} = \frac{1}{\varepsilon} (x_1 g(x_1) - h(x_1) y_1) + k(x_2 - x_1) \\ \frac{dy_1}{dt} = (h(x_1) - m) y_1 + k(y_2 - y_1) \\ \frac{dx_2}{dt} = \frac{1}{\varepsilon} (x_2 g(x_2) - h(x_2) y_2) + k(x_1 - x_2) \\ \frac{dy_2}{dt} = (h(x_2) - m) y_2 + k(y_1 - y_2) \end{array} \right.$$





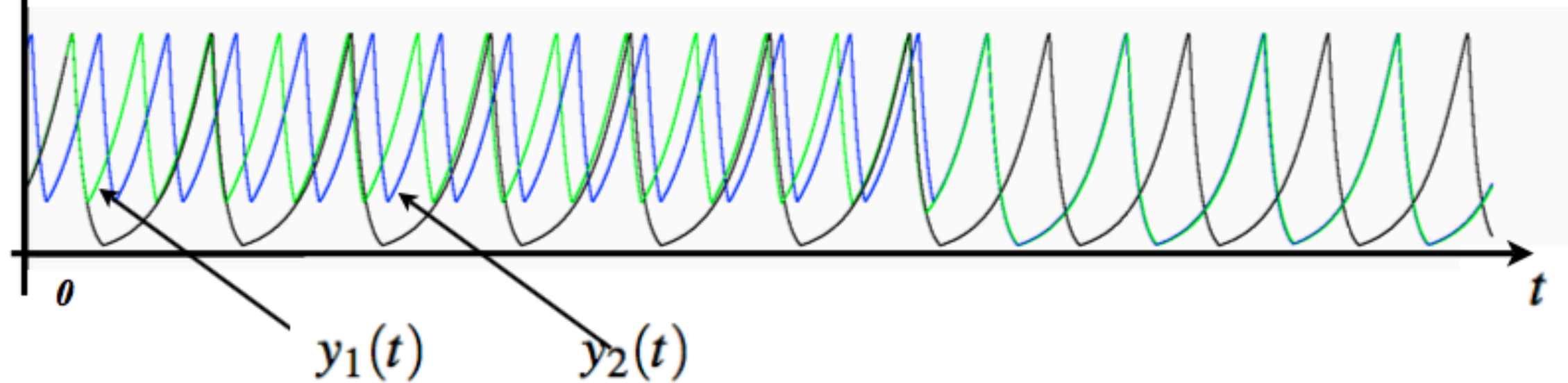
Tant qu'il y a opposition de phase il n'y a pas d'Atto-Fox

$$\left\{ \begin{array}{l} \frac{dx_1}{dt} = \frac{1}{\varepsilon} (x_1 g(x_1) - h(x_1) y_1) + k(x_2 - x_1) \\ \frac{dy_1}{dt} = (h(x_1) - m) y_1 + k(y_2 - y_1) \\ \frac{dx_2}{dt} = \frac{1}{\varepsilon} (x_2 g(x_2) - h(x_2) y_2) + k(x_1 - x_2) \\ \frac{dy_2}{dt} = (h(x_2) - m) y_2 + k(y_1 - y_2) \end{array} \right.$$



Malheureusement !

Le déphasage n'est pas stable



3.3.2. Two patches with threshold migration

In this section we compare periodic solutions of the model (15) of the previous section with solutions of the model :

$$\left\{ \begin{array}{l} \frac{dx_1}{dt} = \frac{1}{\varepsilon}(x_1 g(x_1) - \mu(x_1)y_1) + \nu(x_2) - \nu(x_1) \\ \frac{dy_1}{dt} = (\mu(x_1) - m)y_1 \\ \frac{dx_2}{dt} = \frac{1}{\varepsilon}(x_2 g(x_2) - \mu(x_2)y_2) + \nu(x_1) - \nu(x_2) \\ \frac{dy_2}{dt} = (\mu(x_2) - m)y_2 \end{array} \right. \quad (16)$$

with :

$$- \varepsilon \approx 0$$

$$- g(x) = 2(1 - x)$$

$$- \mu(x) = \frac{x}{0.1 + x}$$

$$- m = 0.7$$

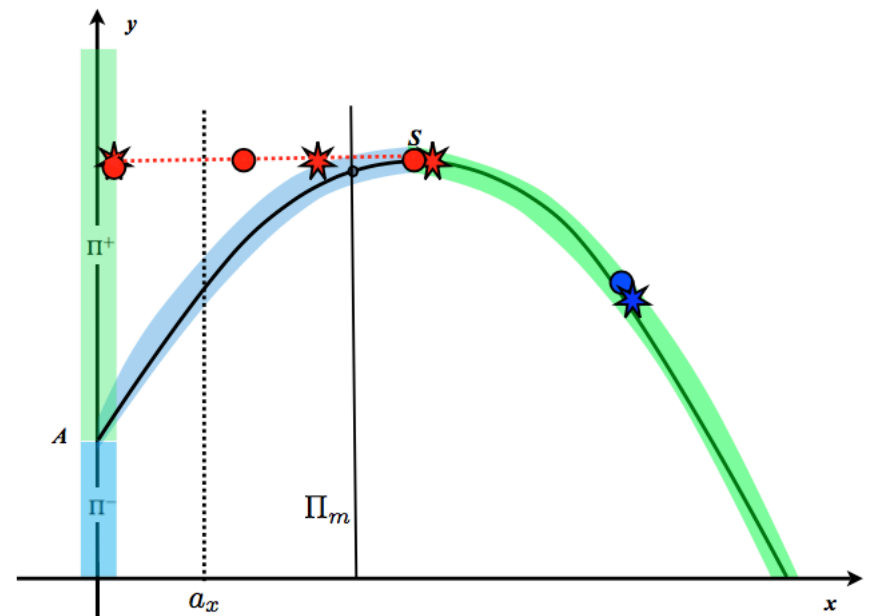
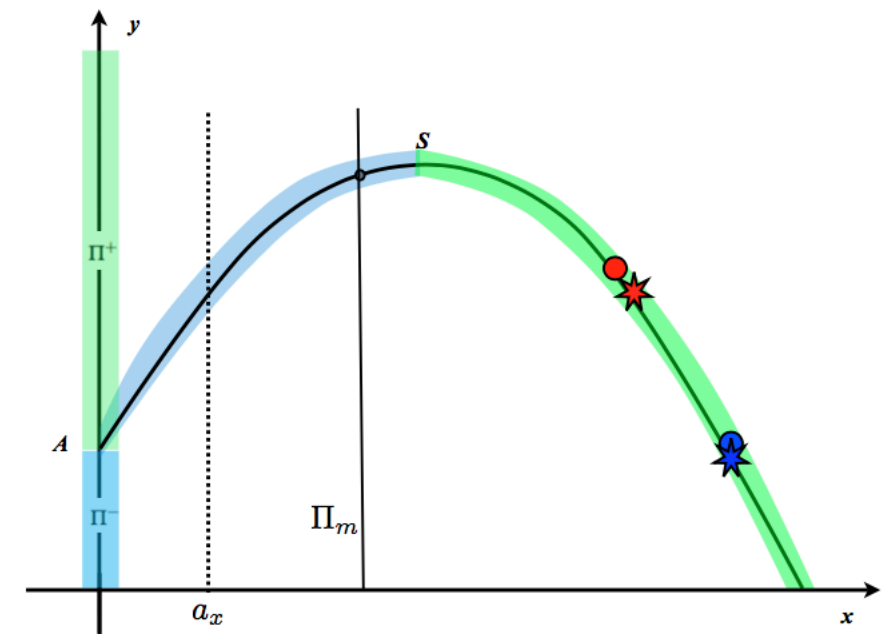
$$- \nu(x) = I \text{ if } x > a = 0.2 \text{ else } \nu = 0 \text{ with } I \text{ **limited**.}$$

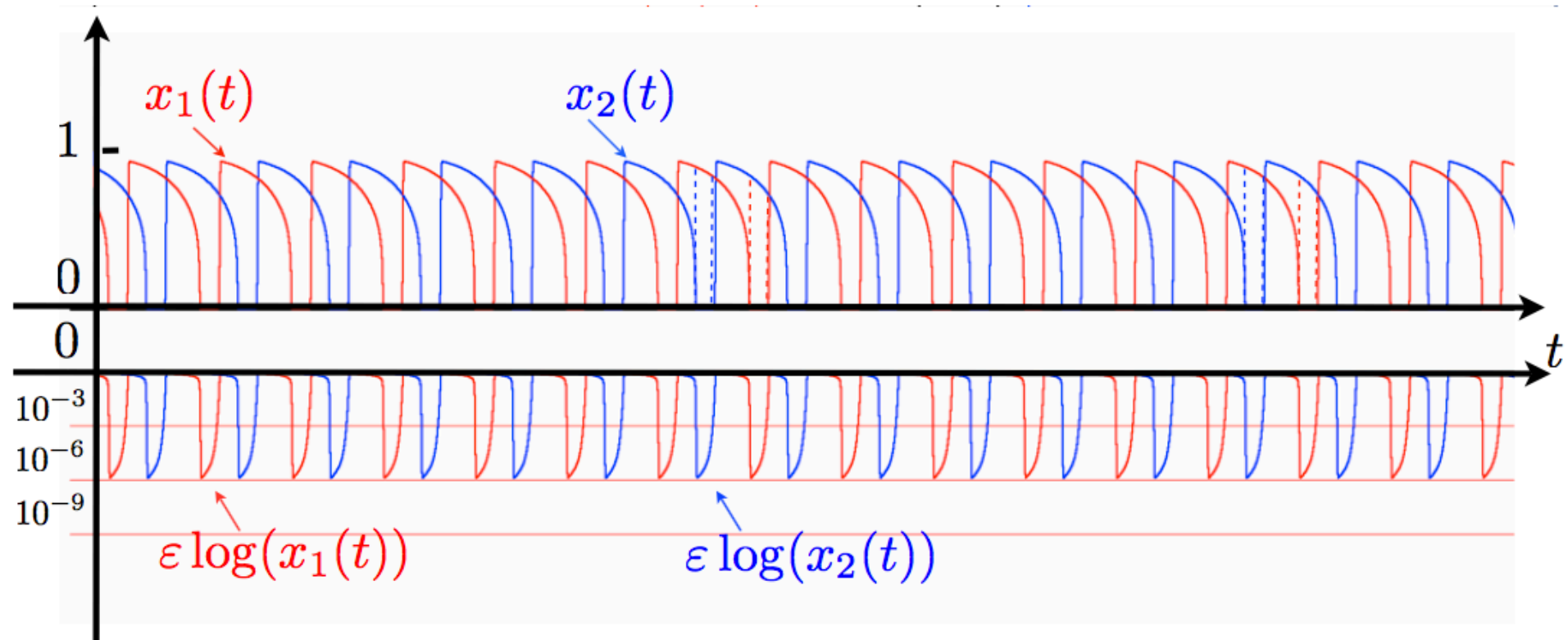
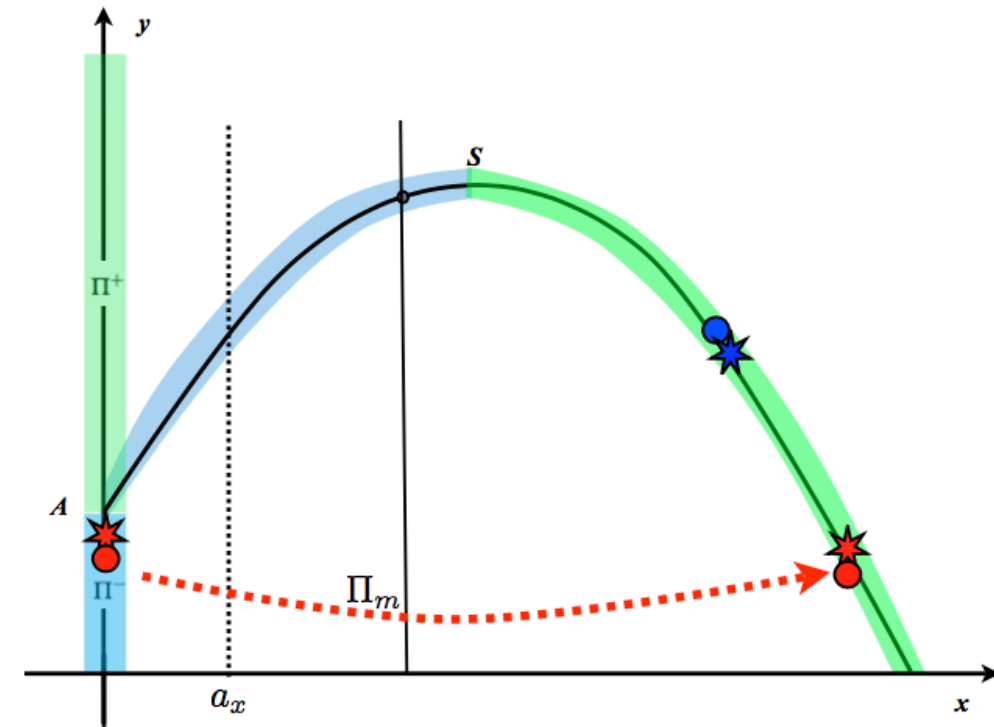
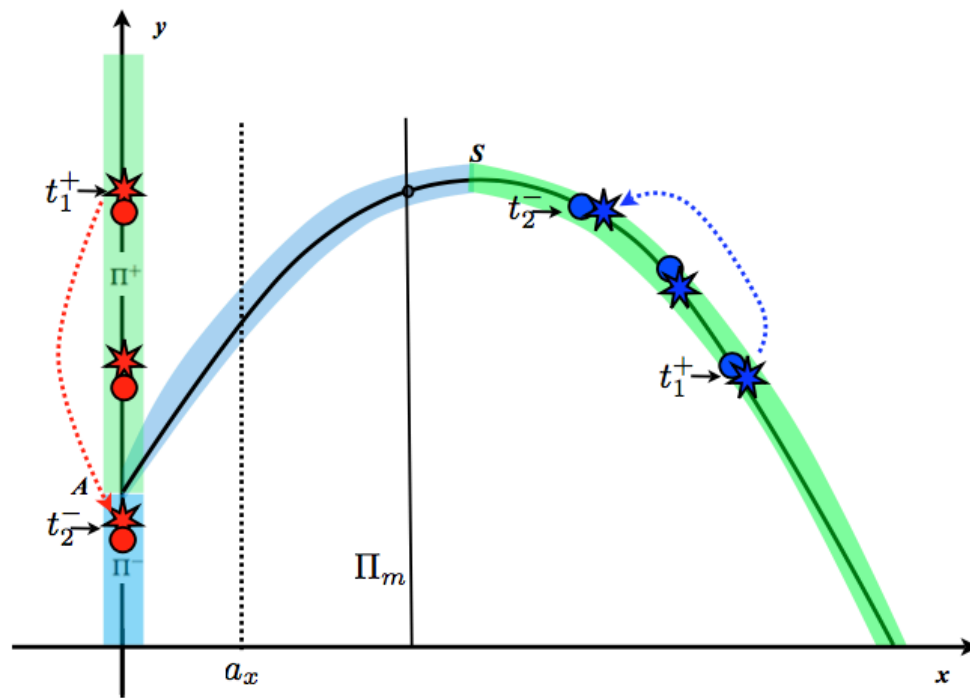
Since there is no migration with y we have simplified the notation ν_x to simply ν . We prove that if an initial condition for system (16) is such that the corresponding solution of (15) is in *phase-opposition* then the solution of (16) will remain **close** to the solution of (15) *during a long transient*. Precisely :

$$\begin{cases} \frac{dx}{dt} = \frac{1}{\varepsilon} \left(2x(1-x) - \frac{x}{0.1+x} y \right) + I \\ \frac{dy}{dt} = \left(\frac{x}{0.1+x} - m \right) y \end{cases}$$

$$\begin{cases} \frac{d\hat{x}_1}{dt} = \frac{1}{\varepsilon} (\hat{x}_1 g(\hat{x}_1) - \mu(\hat{x}_1) \hat{y}_1) + I \\ \frac{d\hat{y}_1}{dt} = (\mu(\hat{x}_1) - m) \hat{y}_1 \\ \frac{d\hat{x}_2}{dt} = \frac{1}{\varepsilon} (\hat{x}_2 g(\hat{x}_2) - \mu(\hat{x}_2) \hat{y}_2) + I \\ \frac{d\hat{y}_2}{dt} = (\mu(\hat{x}_2) - m) \hat{y}_2 \end{cases}$$

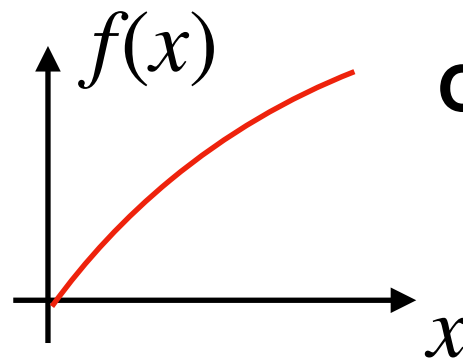
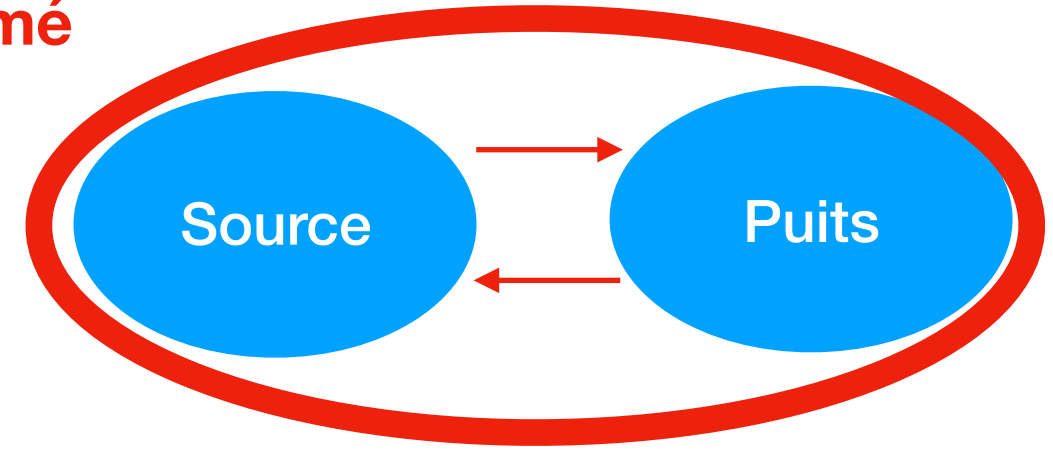
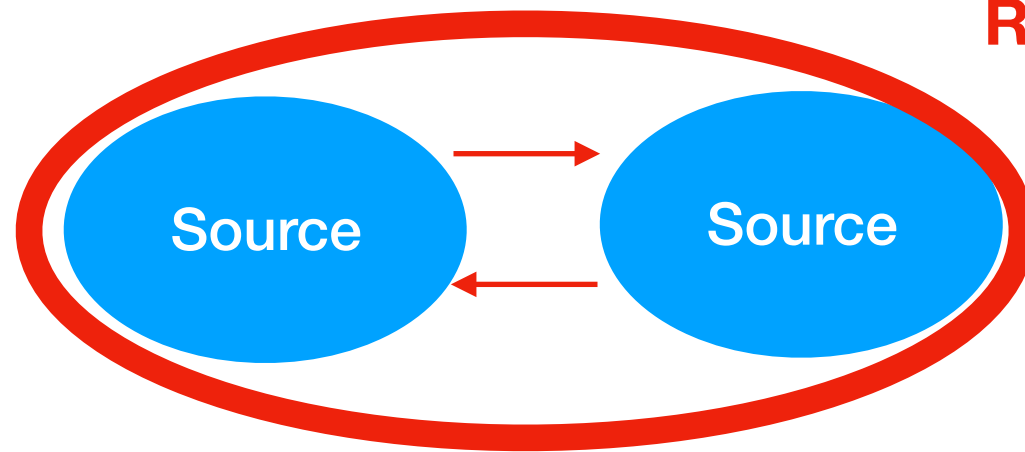
$$\begin{cases} \frac{dx_1}{dt} = \frac{1}{\varepsilon} (x_1 g(x_1) - \mu(x_1) y_1) + \nu(x_2) - \nu(x_1) \\ \frac{dy_1}{dt} = (\mu(x_1) - m) y_1 \\ \frac{dx_2}{dt} = \frac{1}{\varepsilon} (x_2 g(x_2) - \mu(x_2) y_2) + \nu(x_1) - \nu(x_2) \\ \frac{dy_2}{dt} = (\mu(x_2) - m) y_2 \end{cases}$$





Le modèle proie-prédateur en environnement constant

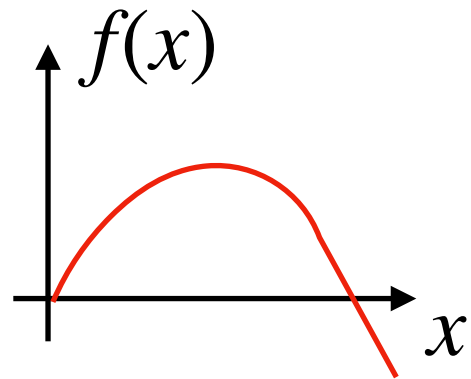
Résumé



Croissance intrinsèque

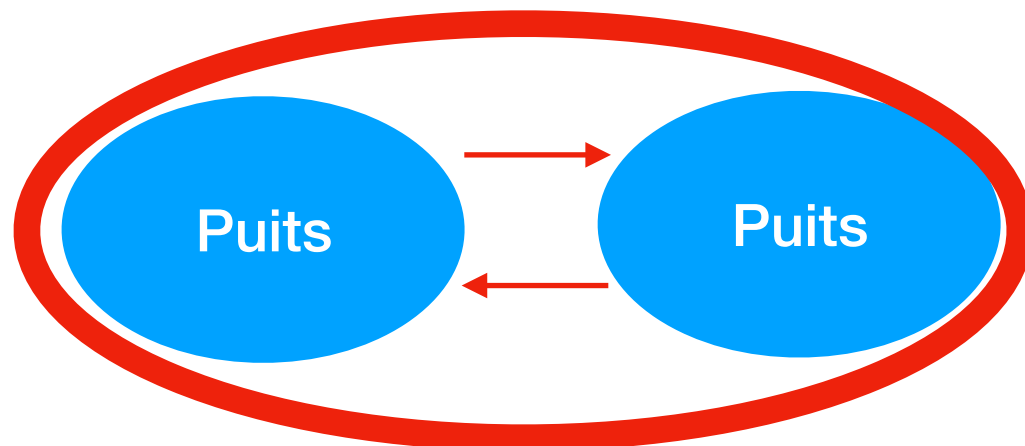
$$\frac{dx}{dt} = rx$$

Rien de spectaculaire



$$\frac{dx}{dt} = rx \left(1 - \frac{x}{K} \right)$$

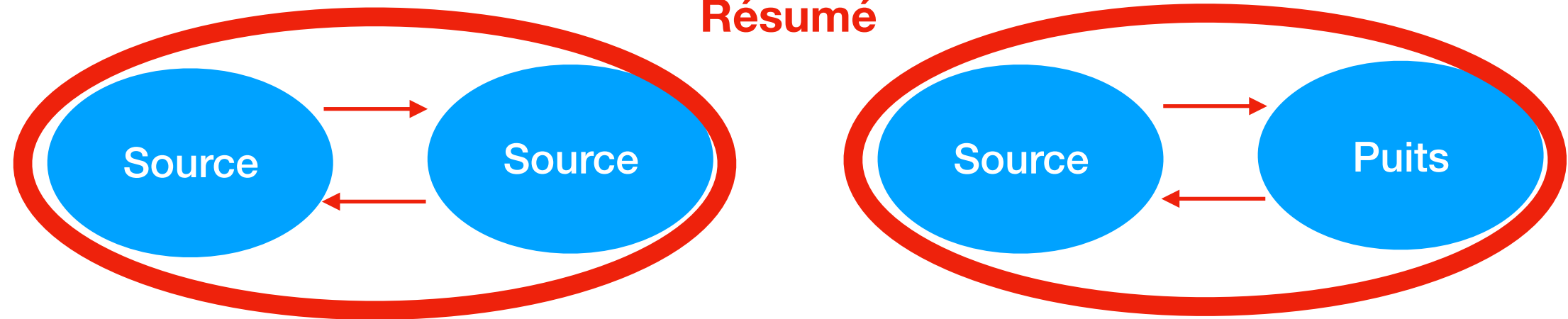
**La TEP dépend de façon significative
et pas toujours intuitive de la migration**



Toujours extinction

Le modèle proie-prédateur en environnement constant

Résumé

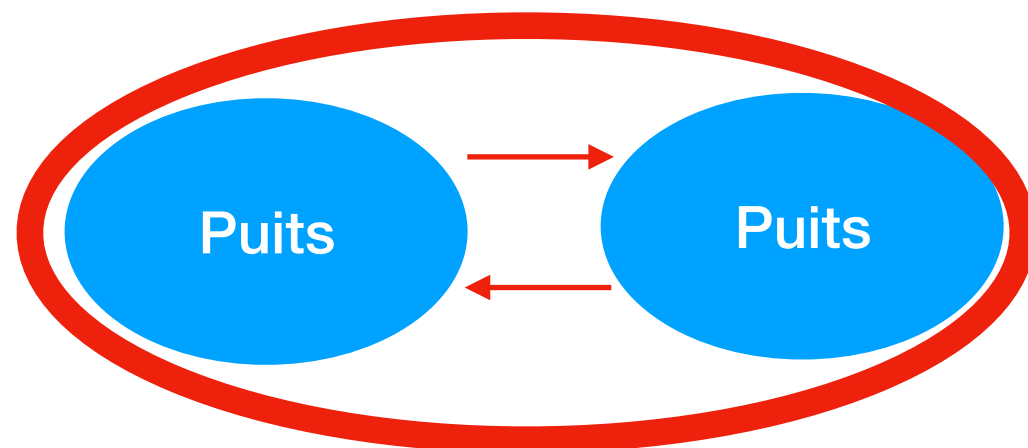


Proc. Natl. Acad. Sci. USA
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Ecology

Populations can persist in an environment consisting of sink habitats only

VINCENT A. A. JANSEN^{*†‡} AND JIN YOSHIMURA^{*§¶}

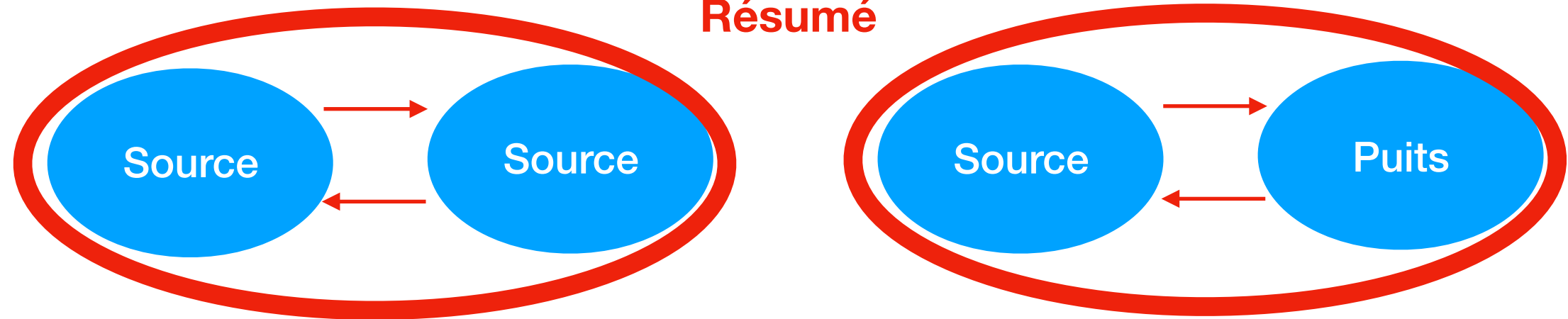
^{*}NERC Centre for Population Biology, Imperial College at Silwood Park, Ascot, Berkshire SL5 7PY, United Kingdom; [§]Department of Systems Engineering, Shizuoka University, 3-5-1 Johoku, Hamamatsu, 432 Japan; and [¶]Department of Environmental and Forest Biology, State University of New York, College of Environmental Science and Forestry, Syracuse, NY 13210



Toujours extinction

Le modèle proie-prédateur en environnement constant

Résumé



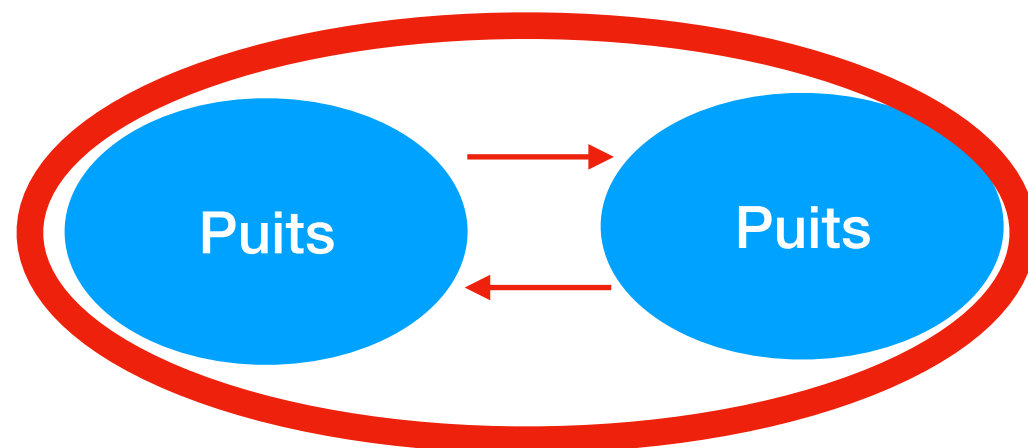
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En environnement variable

Populations can persist in an environment consisting of sink habitats only

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Toujours extinction

Le modèle proie-prédateur en environnement constant