Les modèles du chémostat considérés du point de vue épidémiologique

C. Lobry

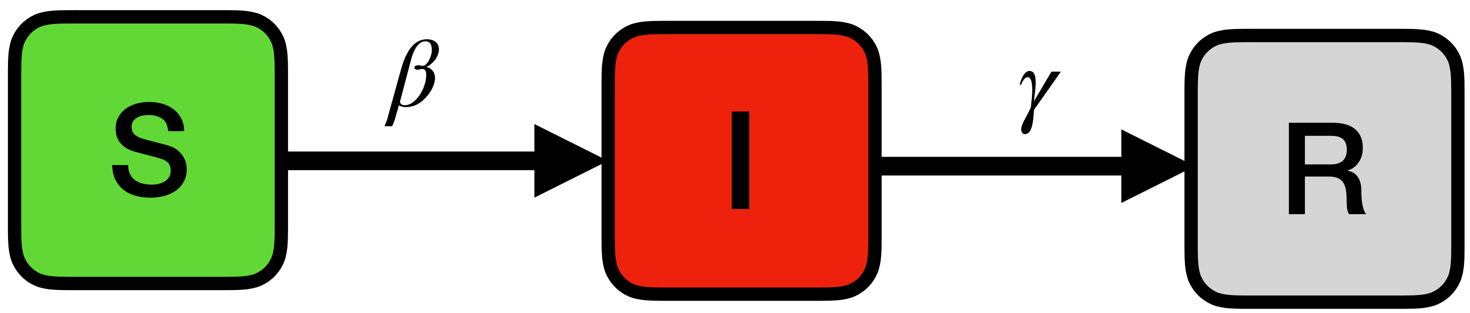
Ecole CIMPA Tunisie 220924-221003

~~Les modèles du chémostat considérés du point de vue épidemiologique~~

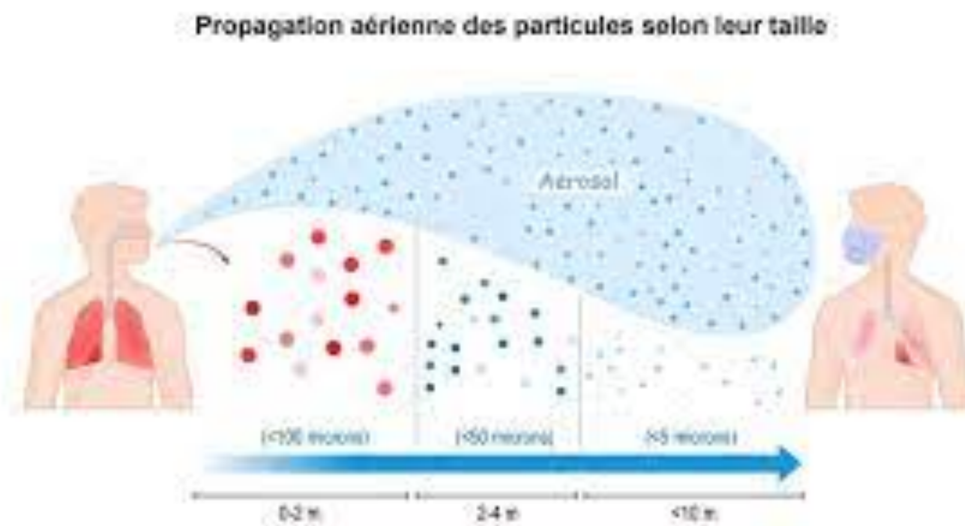
C. Lobry

Modèles de croissance et migration

Le modèle Susceptibles-Infectés-Retirés

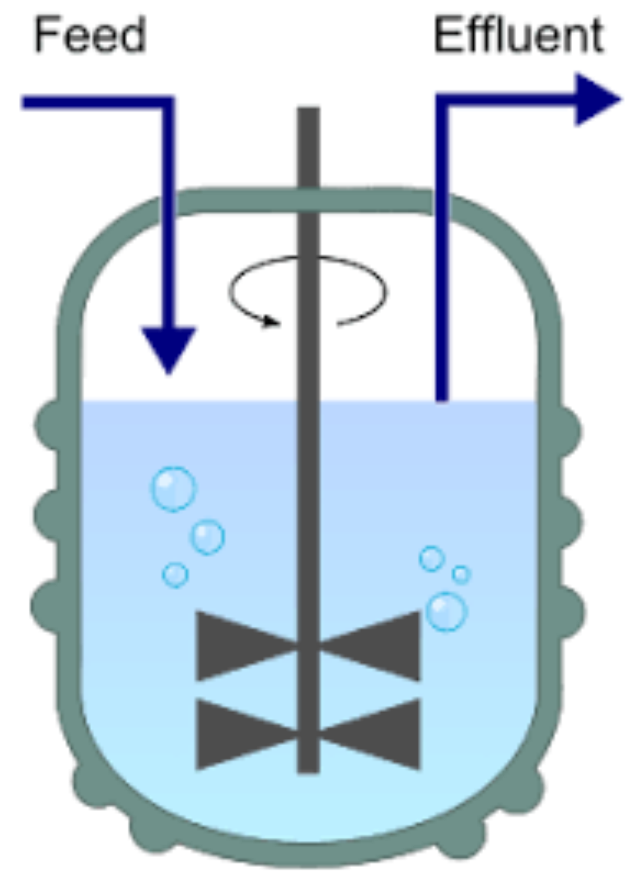


$$\begin{aligned}\frac{dS}{dt} &= -\beta SI \\ \frac{dI}{dt} &= \beta SI - \gamma I \\ \frac{dR}{dt} &= \gamma R\end{aligned}$$



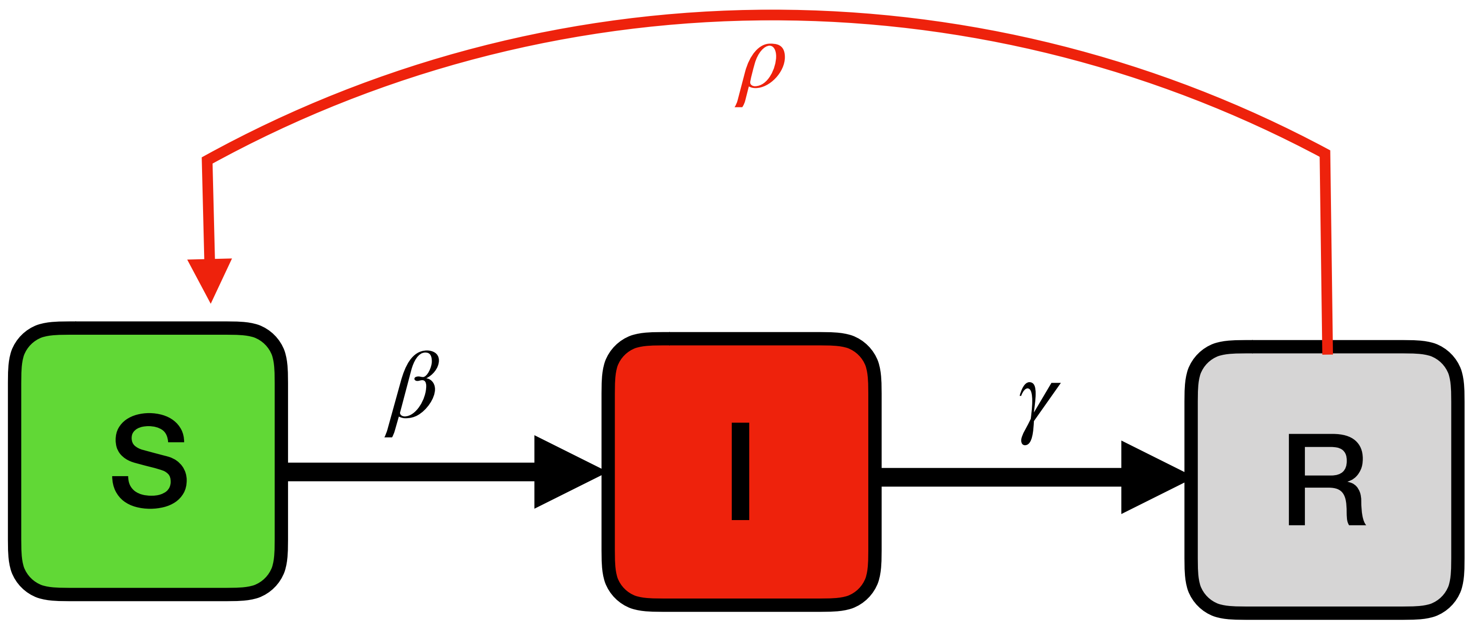
$$S \xrightarrow{\beta} I \xrightarrow{\gamma} R$$

$$\begin{aligned} \frac{dS}{dt} &= -\beta S I \\ \frac{dI}{dt} &= \beta S I - \gamma I \\ \frac{dR}{dt} &= \gamma R \end{aligned}$$



$$\begin{aligned} \frac{dS}{dt} &= D(S_{in} - S) - \mu(S)X \\ \frac{dX}{dt} &= (\mu(S) - D)X \end{aligned}$$

Chémostat



$$
\begin{aligned}
\frac{dS}{dt} &= -\beta S I + \rho R \\
\frac{dI}{dt} &= \beta S I - \gamma I \\
\frac{dR}{dt} &= \gamma R - \rho R
\end{aligned}
$$

$$\begin{aligned}
\frac{dS}{dt} &= -\beta SI + \rho R \\
\frac{dI}{dt} &= \beta SI - \gamma I \\
\frac{dR}{dt} &= \gamma R - \rho R
\end{aligned}$$

$$S + I + R = K$$

$$\begin{aligned}
\frac{dS}{dt} &= \rho K - (\beta SI + \rho(S + I)) \\
\frac{dI}{dt} &= \beta SI - \gamma I
\end{aligned}$$

$$\begin{aligned}
\frac{dS}{dt} &= DS_{in} - (\mu(S)X + DS) \\
\frac{dX}{dt} &= \mu(S)X - \Delta X
\end{aligned}$$

$$\frac{dS}{dt} = -\beta SI + \rho R$$
$$\frac{dI}{dt} = \beta SI - \gamma I$$
$$\frac{dR}{dt} = \gamma R - \rho R$$

Loi d'action de masse

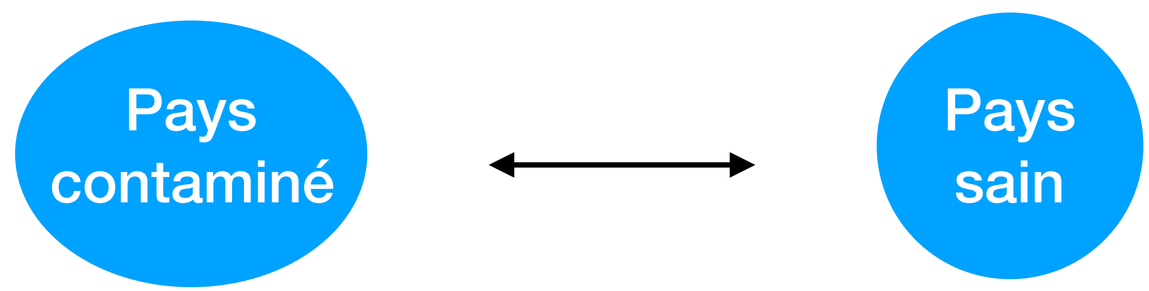
$$\mu(S)X \sim \mu \, . \, S \, . \, X \Longleftrightarrow \beta \, . \, S \, . \, I$$

$$S + I + R = K$$

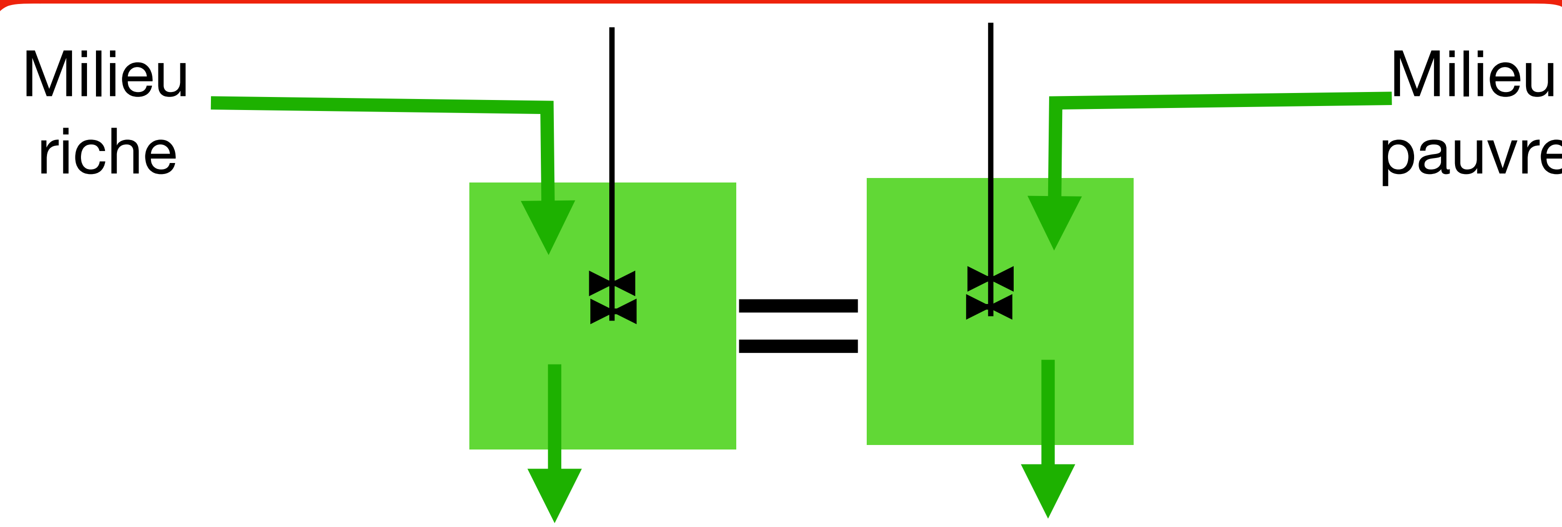
$$\frac{dS}{dt} = \rho K - (\beta SI + \rho(S + I))$$
$$\frac{dI}{dt} = \beta SI - \gamma I$$

$$\frac{dS}{dt} = DS_{in} - (\mu(S)X + DS)$$
$$\frac{dX}{dt} = \mu(S)X - \Delta X$$

**Du point de vue des modèles mathématiques
il y a peu de différences entre un modèle de croissance de population
et un modèle de développement d'une épidémie**



Mesures de confinement et développement de l'infection



Chémostats interconnectés

Cours CIMPA

Lecture notes on mathematical models of interconnected chemostats

Alain Rapaport

▶ To cite this version:

> Alain Rapaport. Lecture notes on mathematical models of interconnected chemostats. École thématique. Ecole de recherche du CIMPA, Kenitra, Morocco. 2019, pp.55. cel-02182956

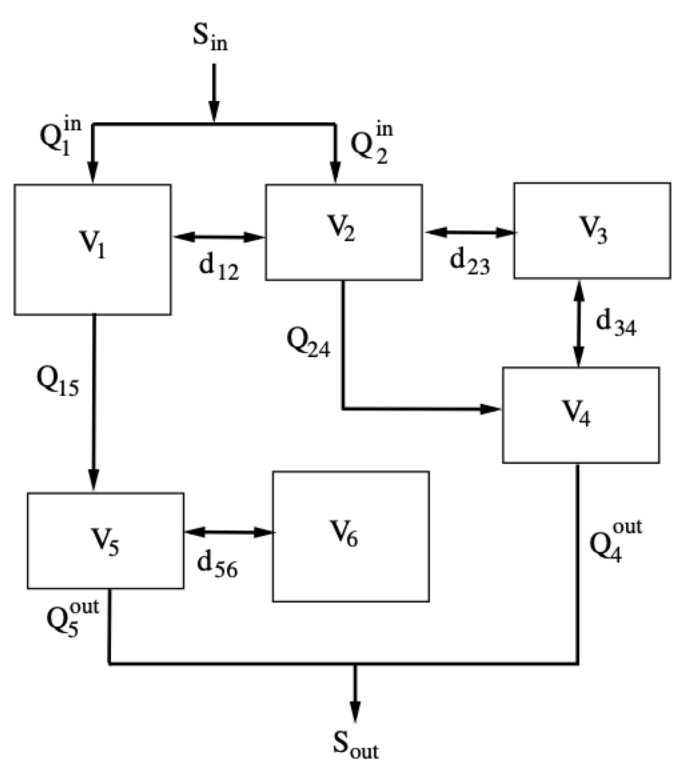


Figure 4: Example of a "general" gradostat

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Lecture notes on mathematical models of interconnected chemostats

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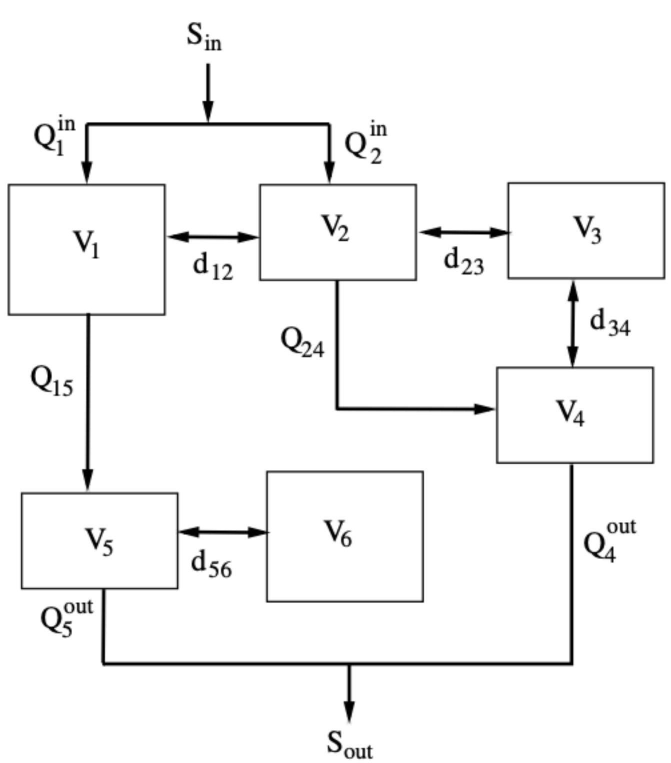

Figure 4: Example of a "general" gradostat

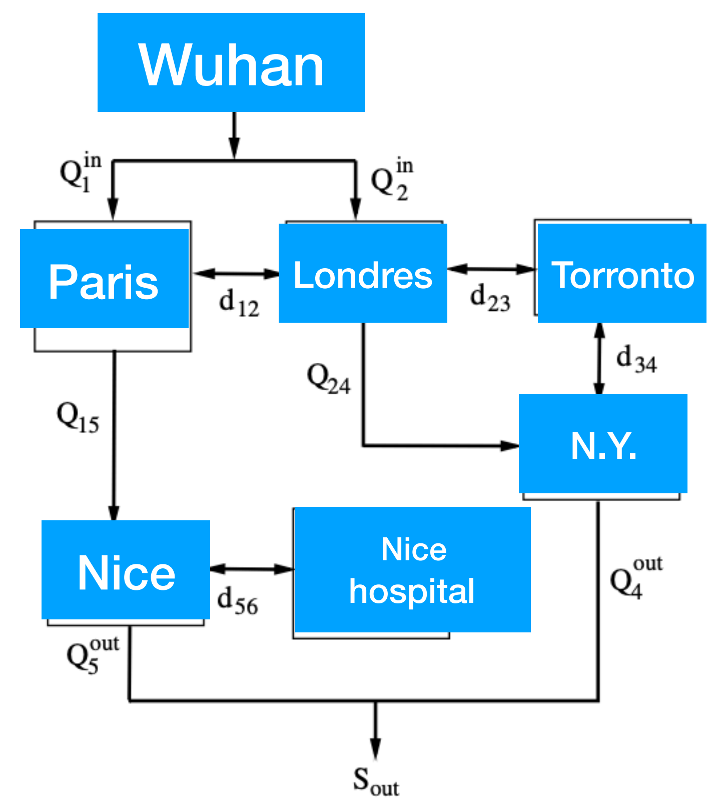

Figure 4: Example of a "general" gradostat


COLLECTION GÉNIE DES PROCÉDÉS
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Jérôme Harmand, Claude Lobry
Alain Rapaport et Tewfik Sari
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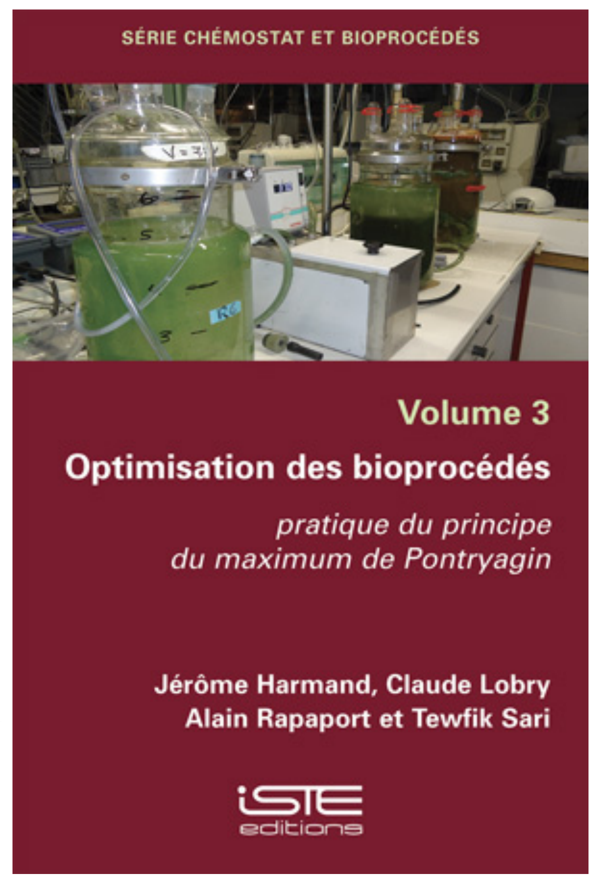
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Série Chémostat et bioprocédés
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Claude Lobry
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Dynamique des populations

Source ⟷ Croissance et migration ⟷ Puits

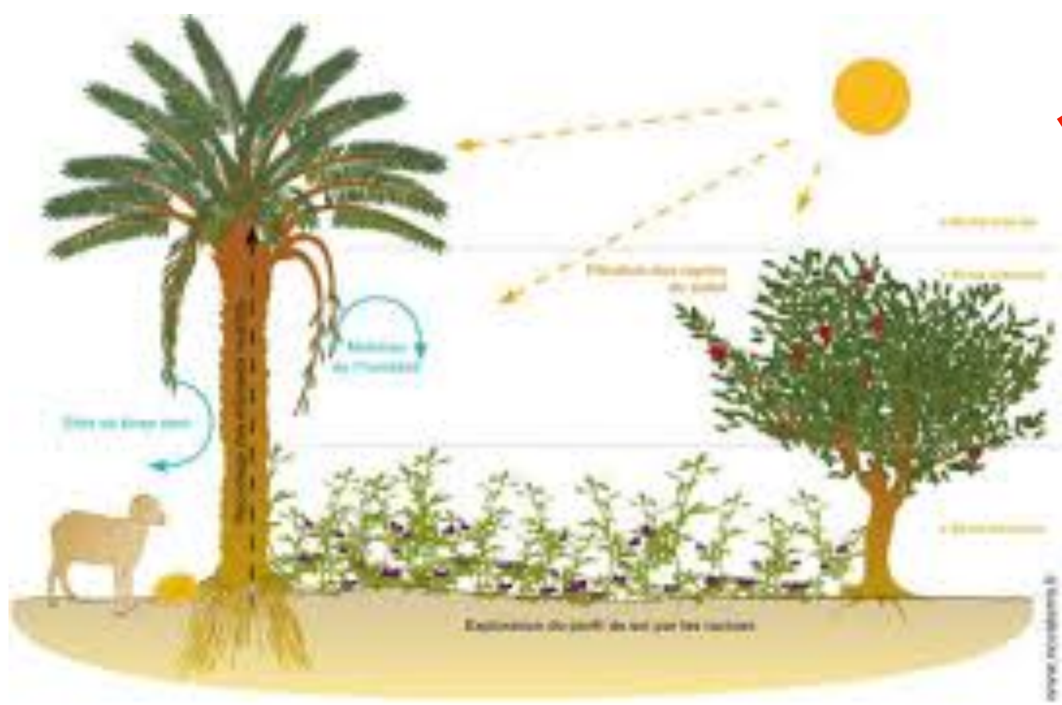

Table 1. Summary characteristics of variations on the source-sink dynamics model.

	Source-sink	Source-pseudosink	Ecological trap
Source patch (high quality habitat)	Stable or growing Attractive Net exporter	Stable or growing Attractive Net exporter	Stable or growing ***Avoided (or equal)*** Net exporter
Sink, pseudo-sink, or trap patch (low quality habitat)	Declines to extinction Avoided Net importer	***Declines to stable size*** Either Net importer	Declines to extinction ***Attractive (or equal)*** Net importer

Google Scholar

Robert D. Holt

Eminent Scholar in Biology, Arthur R. Marshall, Jr., Chair in Ecology, University of Florida
Adresse e-mail validée de ufl.edu

Theoretical ecology evolutionary biology population and community …

SUIVRE

TITRE	CITÉE PAR	ANNÉE
The metacommunity concept: a framework for multi-scale community ecology MA Leibold, M Holyoak, N Mouquet, P Amarasekare, JM Chase, ... Ecology letters 7 (7), 601-613	4324	2004
Trophic downgrading of planet Earth JA Estes, J Terborgh, JS Brashares, ME Power, J Berger, WJ Bond, ... science 333 (6040), 301-306	3261	2011
Predation, apparent competition, and the structure of prey communities RD Holt Theoretical population biology 12 (2), 197-229	2638	1977

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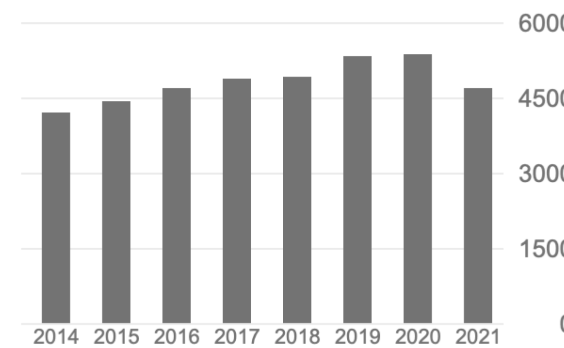


Personnage très important en écologie théorique

Les articles KSBGSH

medRxiv 2020.07.17.

Regional COVID-19 spread despite expected declines: how mitigation is hindered by spatio-temporal variation in local control measures

Nicholas Kortessis[a], Margaret W. Simon[a], Michael Barfield[a], Gregory Glass[b,c], Burton H. Singer[c], Robert D. Holt[a]*

P.N.A.S. 2020.12.01

The interplay of movement and spatiotemporal variation in transmission degrades pandemic control

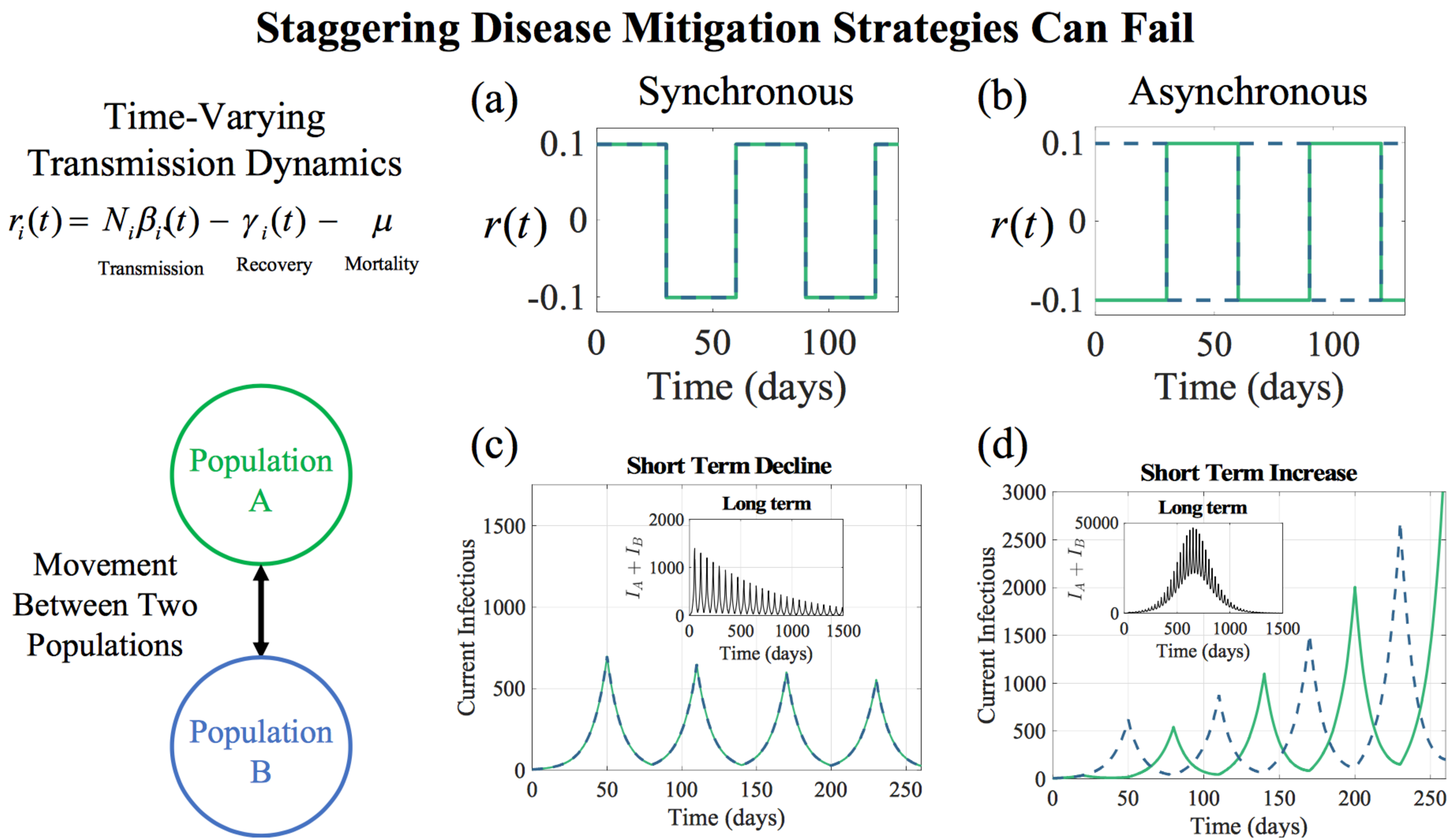
Nicholas Kortessis[a], Margaret W. Simon[a], Michael Barfield[a], Gregory E. Glass[b,c], Burton H. Singer[c], and Robert D. Holt[a,1]

spatiotemporal variation | COVID-19 | inflationary effect

. . . we're a large country that has outbreaks in different regions, different states, different cities, that have different dynamics, and different phases. . .

Anthony Fauci, quoted on CNN, 23 April 2020 (1)

Figures

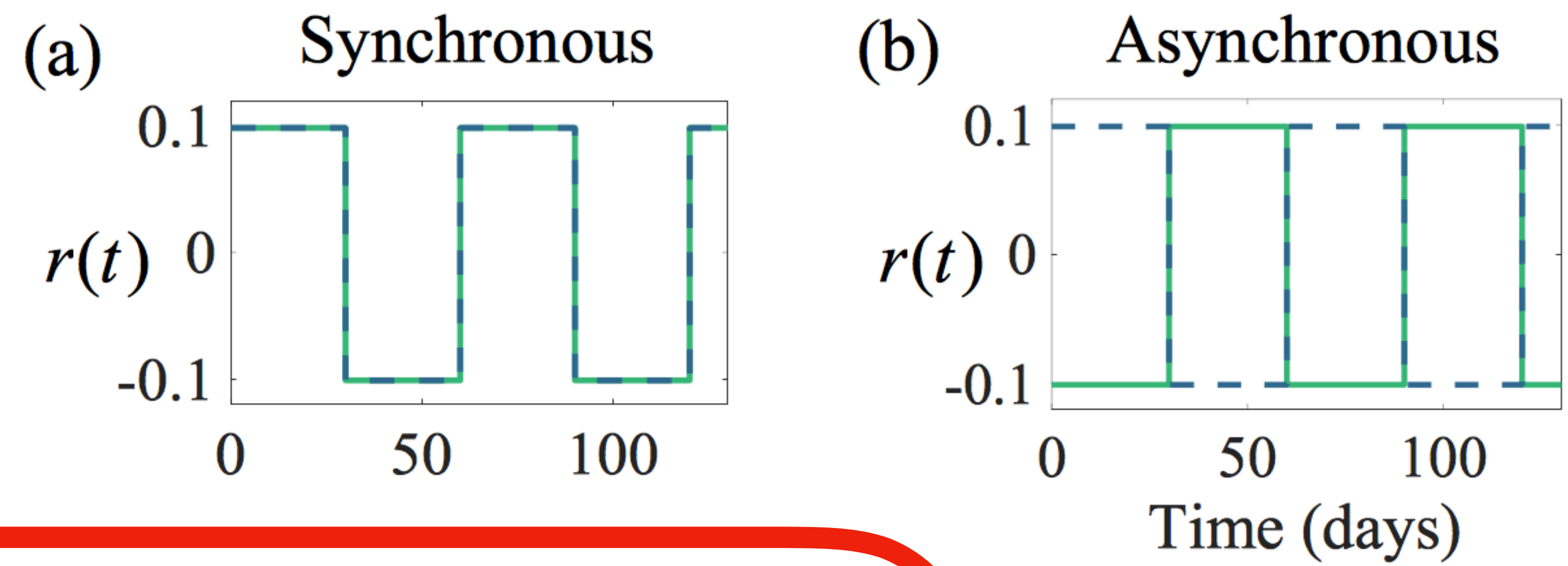

Staggering Disease Mitigation Strategies Can Fail
Time-Varying Transmission Dynamics
$r_i(t) = N_i\beta_i(t) - \gamma_i(t) - \mu$
Transmission
Recovery
Mortality
Movement Between Two Populations
Population A
Population B
(a) Synchronous
r(t)
Time (days)
(b) Asynchronous
(c) Short Term Decline
Long term
Current Infectious
$I_A + I_B$
(d) Short Term Increase

Figures

Staggering Disease Mitigation Strategies Can Fail

Time-Varying
Transmission Dynamics

$$r_i(t) = \underbrace{N_i\beta_i(t)}_{\text{Transmission}} - \underbrace{\gamma_i(t)}_{\text{Recovery}} - \underbrace{\mu}_{\text{Mortality}}$$



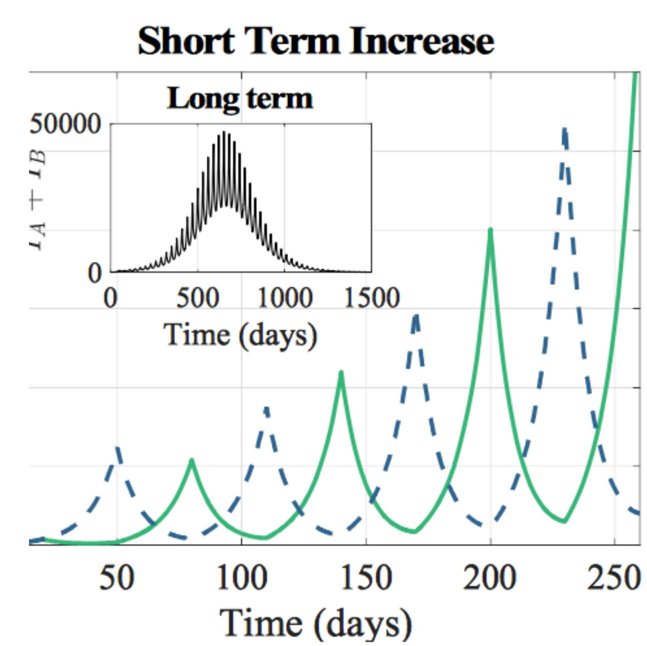
Population Dynamics in Two-Patch Environments: Some Anomalous Consequences of an Optimal Habitat Distribution

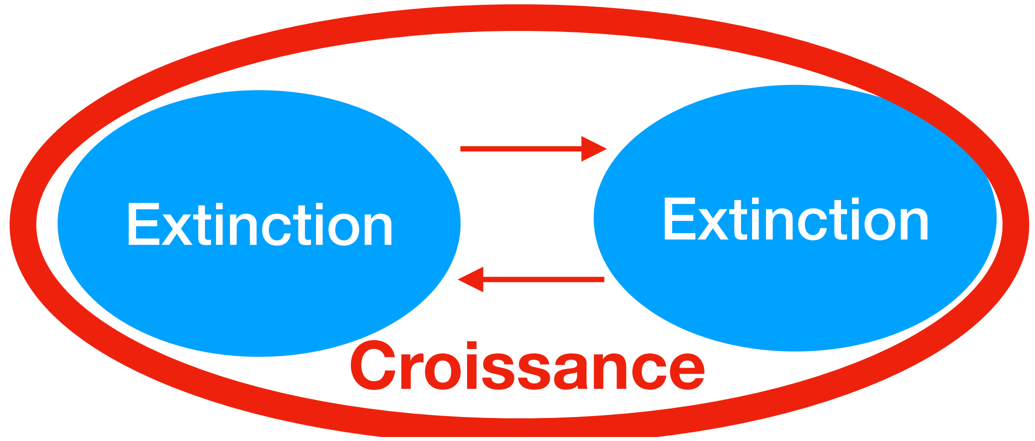
ROBERT D. HOLT

Museum of Natural History and
Department of Systematics and Ecology,
University of Kansas, Lawrence, Kansas 66045

Received January 17, 1984

1984





1998

Proc. Natl. Acad. Sci. USA
Vol. 95, pp. 3696–3698, March 1998
Ecology

Populations can persist in an environment consisting of sink habitats only

VINCENT A. A. JANSEN*†‡ AND JIN YOSHIMURA*§¶

Les mathématiques peuvent (et doivent ?) aider

à clarifier

les concepts

mis en avant par les biologistes

Longue parenthèse historique

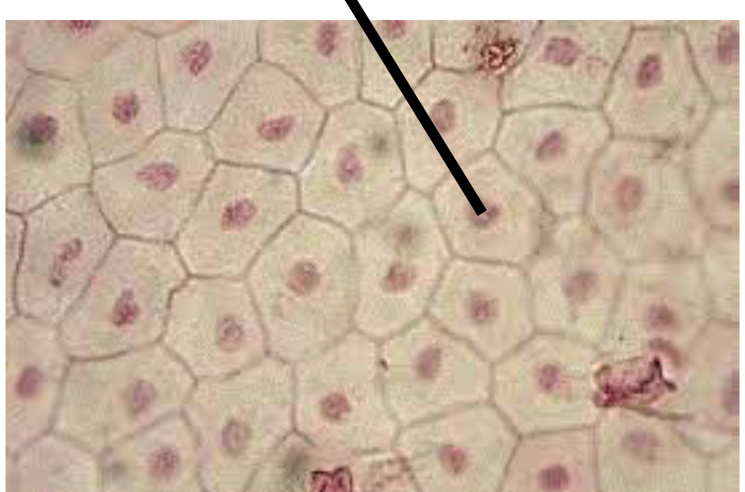
Un exemple :
Les milieux excitables
et les EDR

The Chemical Basis of Morphogenesis

A. M. Turing

Philosophical Transactions of the Royal Society of London. Series B, Biological Sciences, Vol. 237, No. 641 (Aug. 14, 1952), 37-72.

$$\boxed{\begin{aligned}\frac{\mathrm{d}X_r}{\mathrm{d}t} &= f(X_r, Y_r)\\ \frac{\mathrm{d}Y_r}{\mathrm{d}t} &= g(X_r, Y_r)\end{aligned}} \qquad (r = 1, \dots, N)$$



Equations de diffusion-reaction

parenthèse historique

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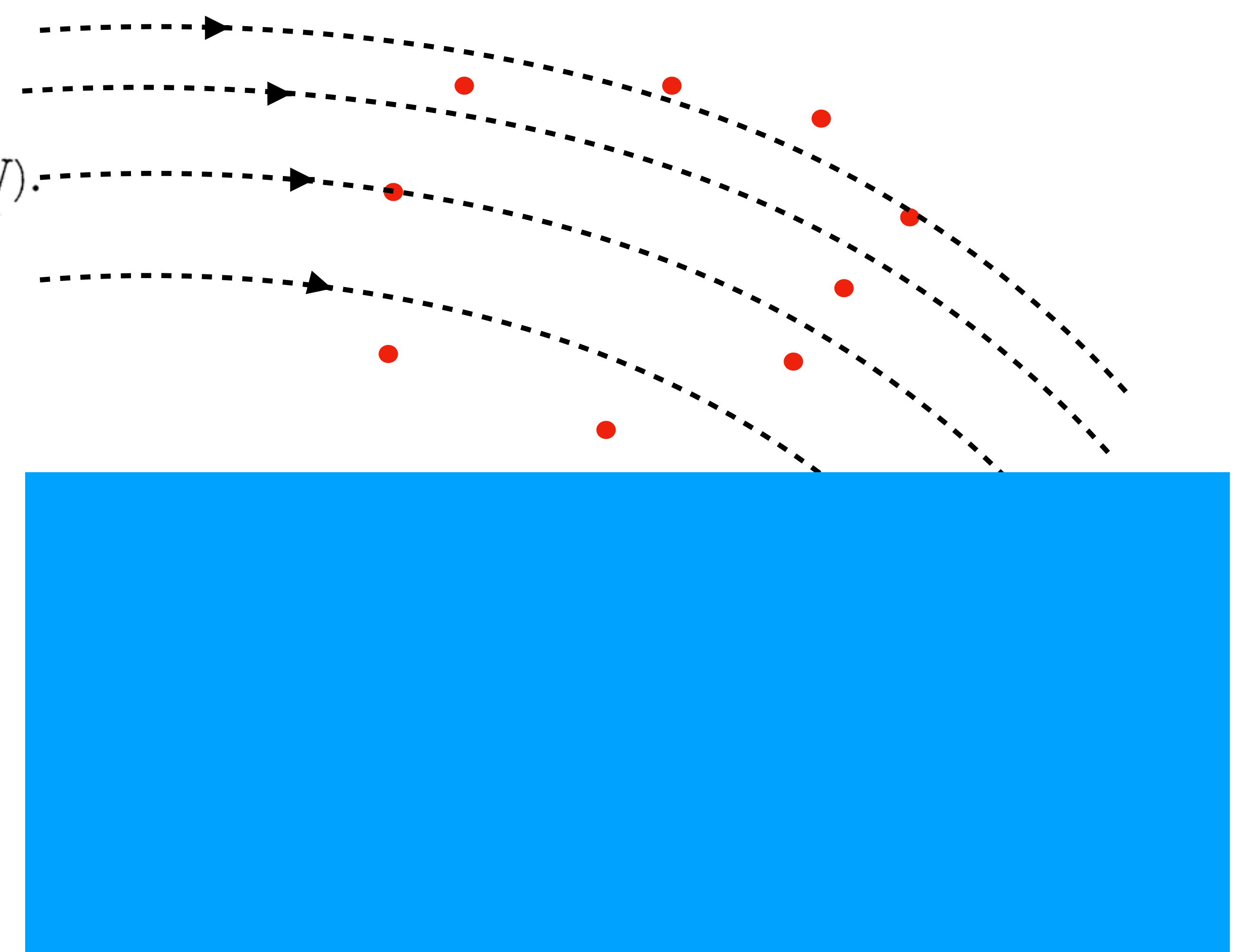

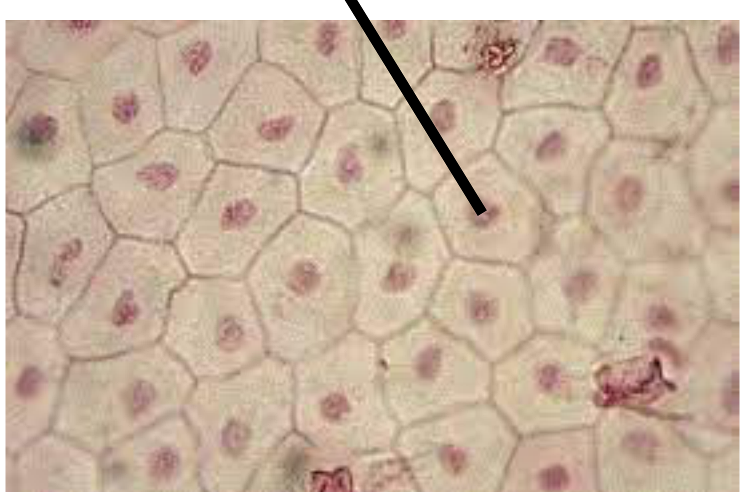

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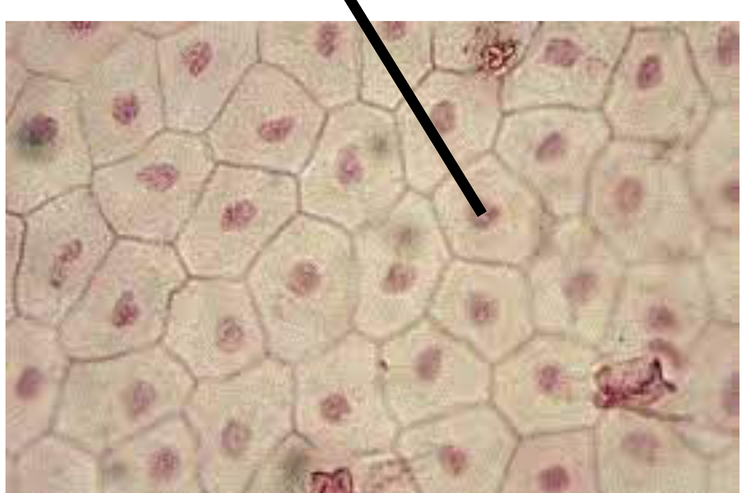
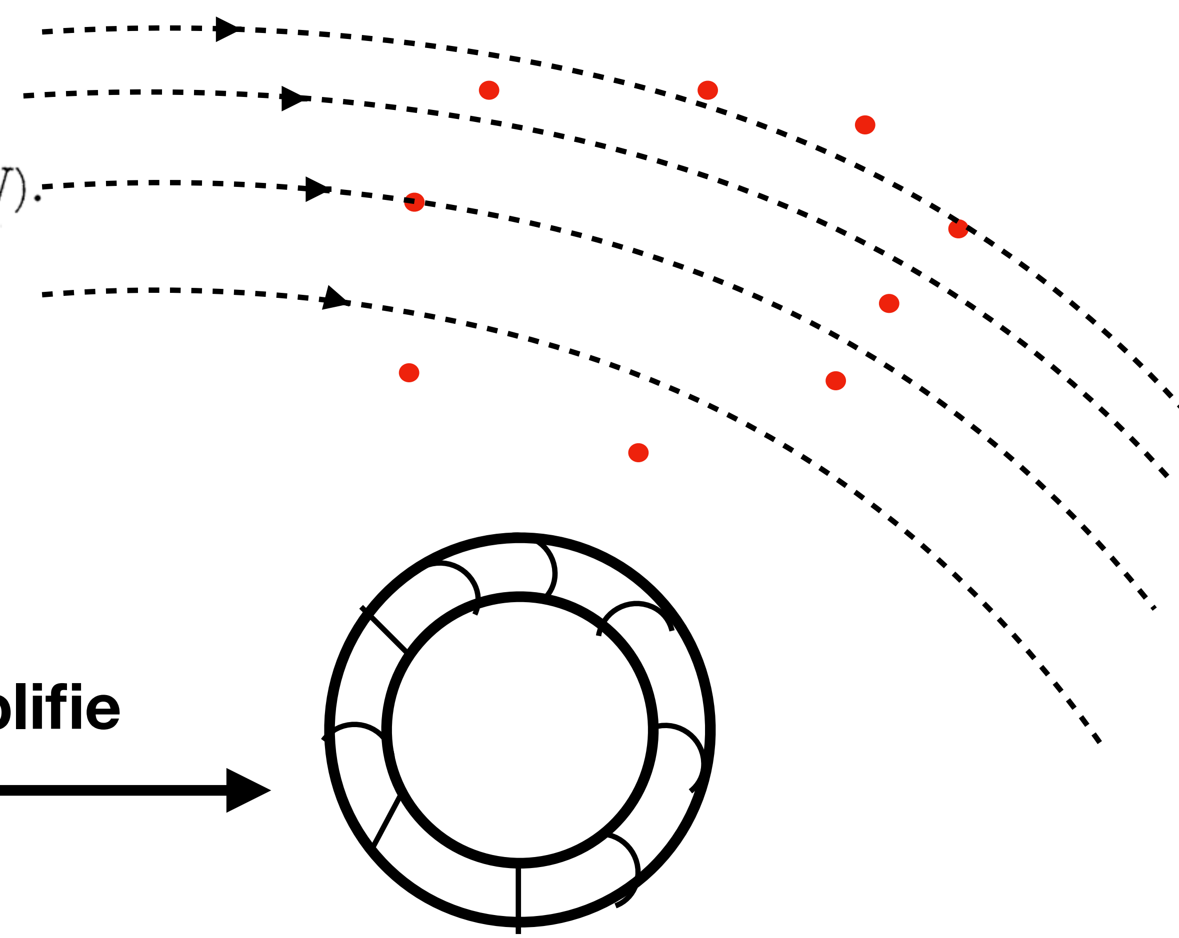


Simplifie



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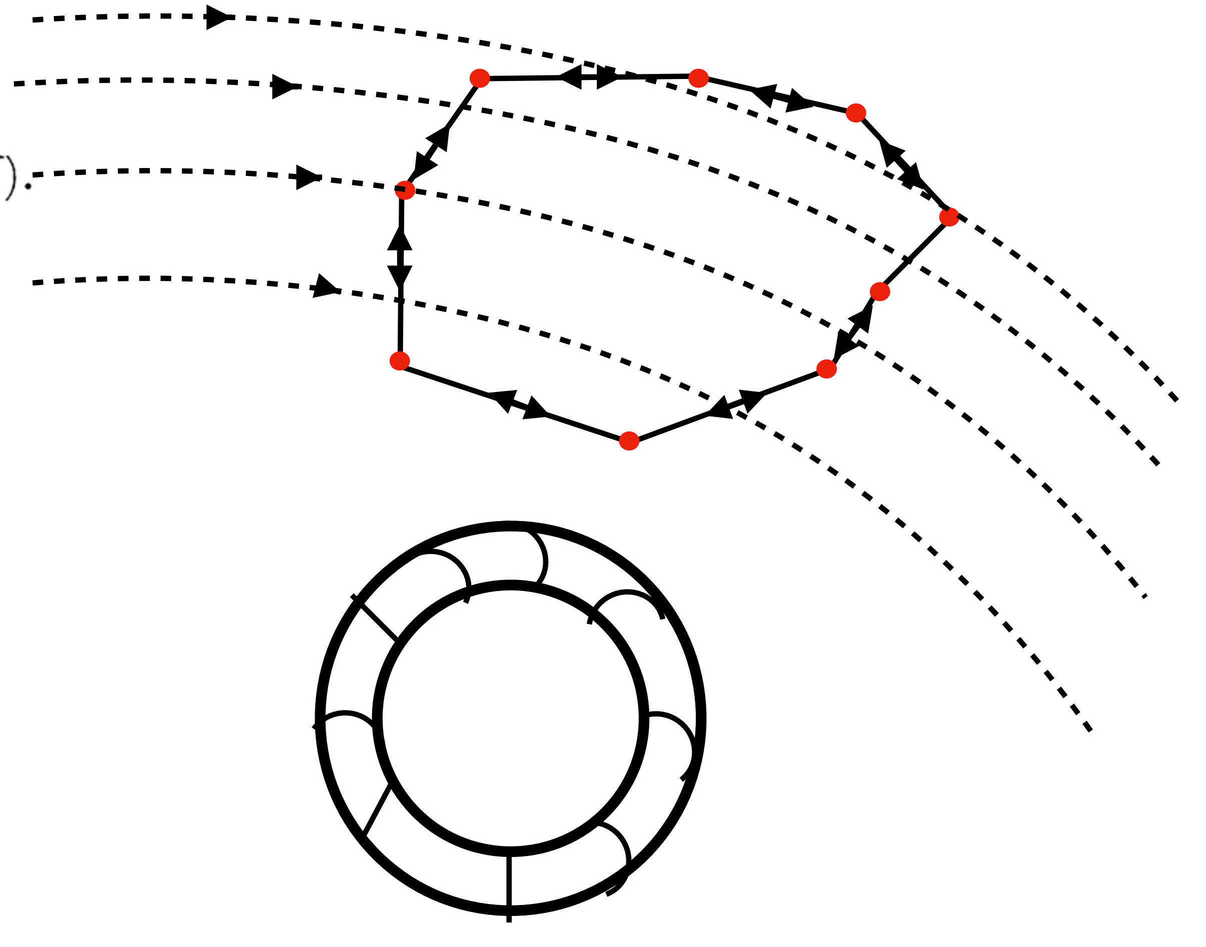
$$\left.\begin{aligned}\frac{\mathrm{d}X_r}{\mathrm{d}t} &= f(X_r, Y_r) + \mu(X_{r+1} - 2X_r + X_{r-1})\\ \frac{\mathrm{d}Y_r}{\mathrm{d}t} &= g(X_r, Y_r) + \nu(Y_{r+1} - 2Y_r + Y_{r-1})\end{aligned}\right\} \quad (r = 1, \ldots, N).$$



Equations de diffusion-reaction

parenthèse historique

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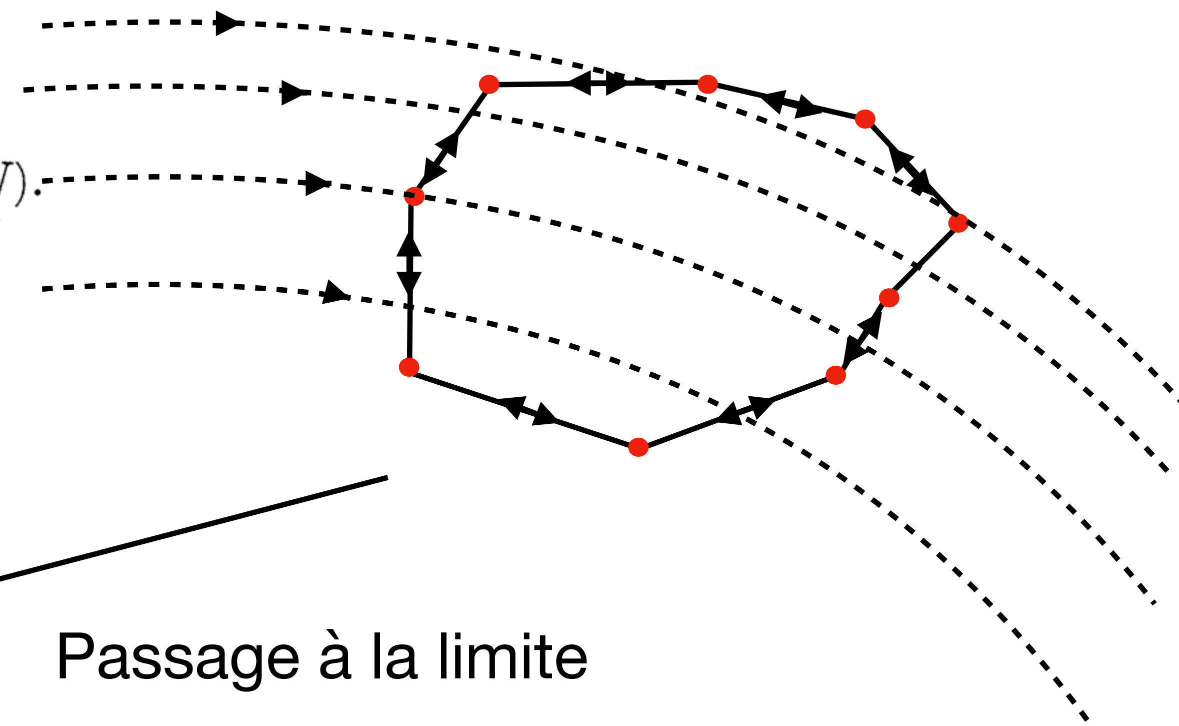

Passage à la limite

$$\left.\begin{aligned}\frac{\partial X}{\partial t} &= a(X-h) + b(Y-k) + \frac{\mu'}{\rho^2}\frac{\partial^2 X}{\partial \theta^2},\\ \frac{\partial Y}{\partial t} &= c(X-h) + d(Y-k) + \frac{\nu'}{\rho^2}\frac{\partial^2 Y}{\partial \theta^2},\end{aligned}\right\}$$

Equations de diffusion-reaction

The Chemical Basis of Morphogenesis

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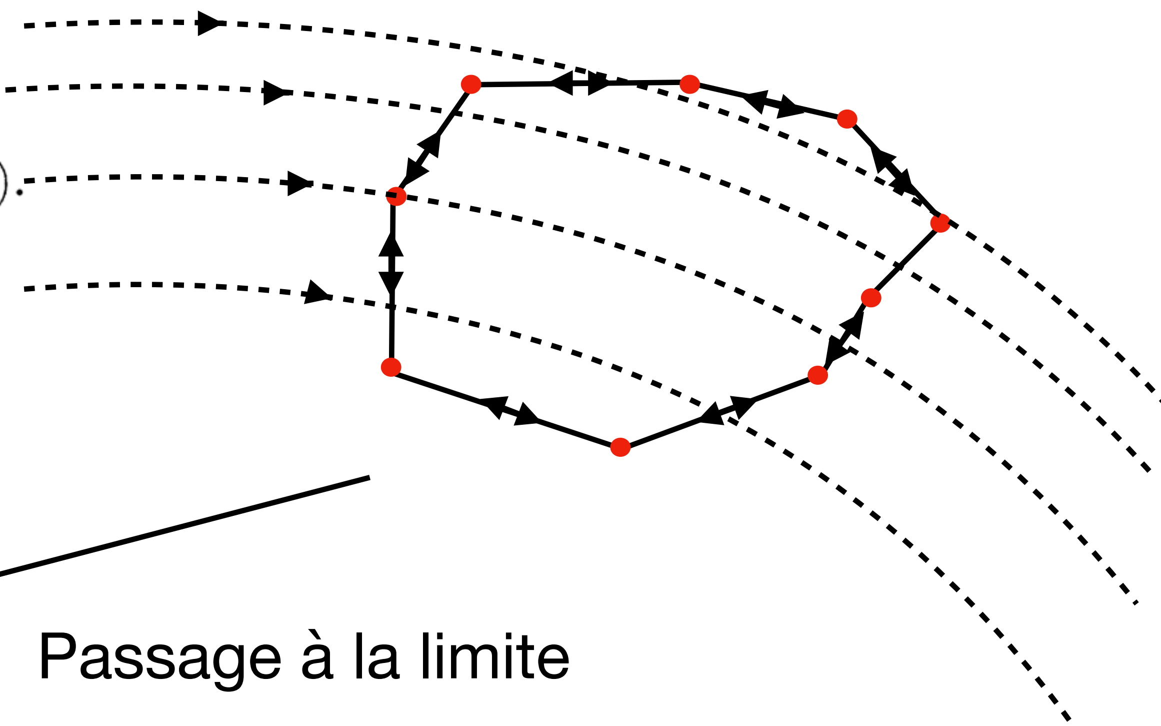

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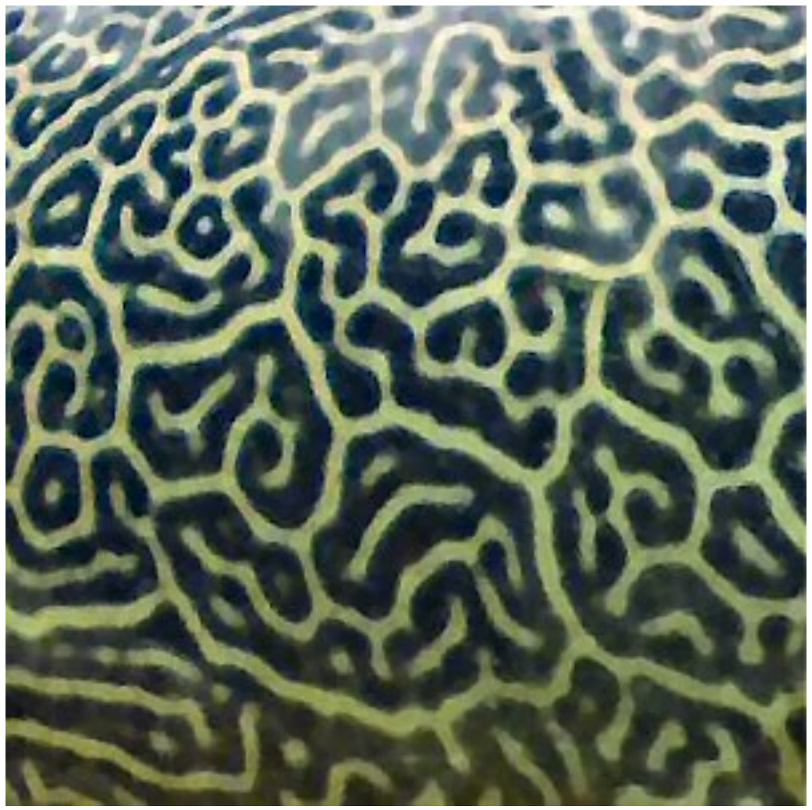
Passage à la limite

Equations de diffusion-reaction

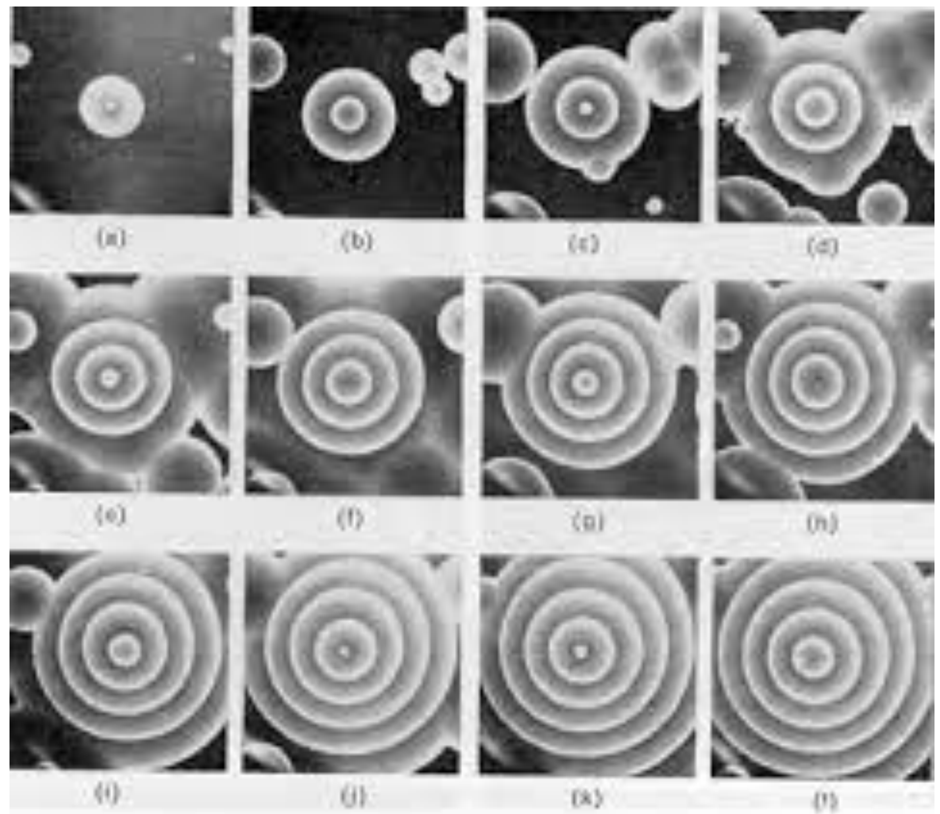
parenthèse historique

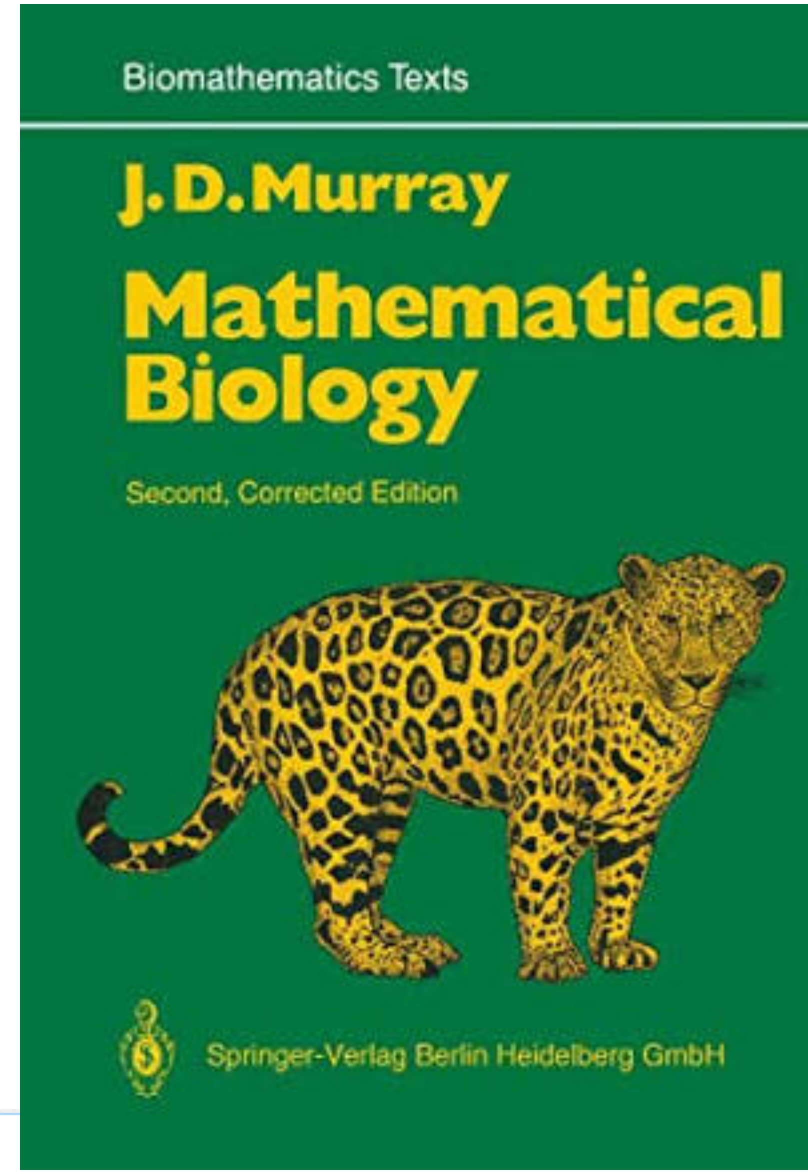
(1980-2000)

$$\frac{\partial U}{\partial t} = f(t,U) + k^2 \Delta U \qquad U = (U_1..U_n)$$



Mbuna poisson-globe





Scholarpedia is supported by Brain Corporation

Belousov-Zhabotinsky reaction

Anatol M. Zhabotinsky (2007), Scholarpedia, 2(9):1435. doi:10.4249/scholarpedia.1435

- **Dr. Anatol M. Zhabotinsky**, Brandeis University, Waltham, MA

Equations de diffusion-reaction

parenthèse historique

On the Spatial Spread of Rabies among Foxes

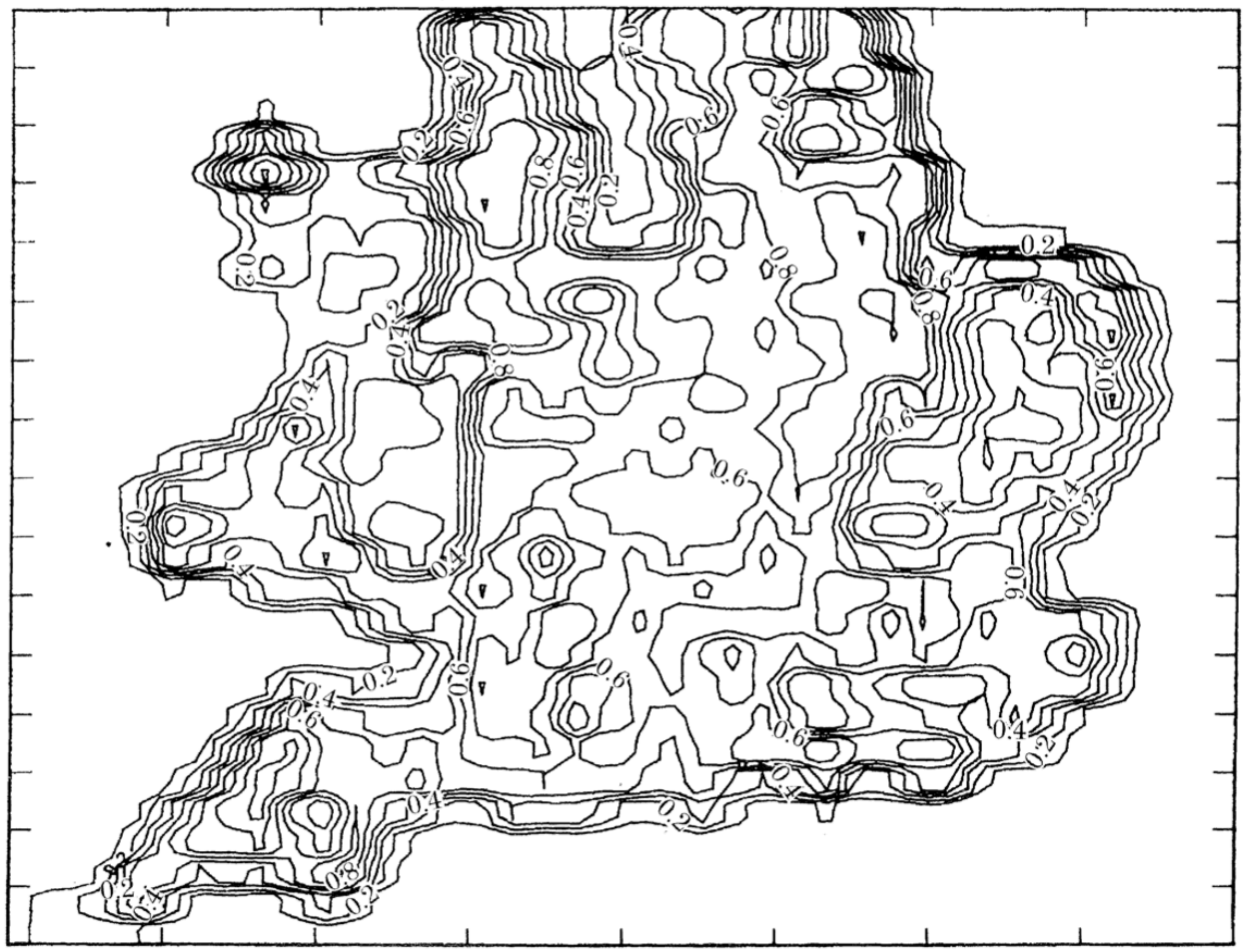
J. D. Murray; E. A. Stanley; D. L. Brown

Proceedings of the Royal Society of London. Series B, Biological Sciences, Vol. 229, No. 1255 (Nov. 22, 1986), 111-150.

$$\partial S/\partial T = (a-b)(1-N/K)S-\beta RS,$$

$$\partial I/\partial T = \beta SR-\sigma I-[b+(a-b)N/K]I,$$

$$\partial R/\partial T = \sigma I-\alpha R-[b+(a-b)N/K]R+D\partial^2 R/\partial X^2,$$



Equations de diffusion-reaction

parenthèse historique

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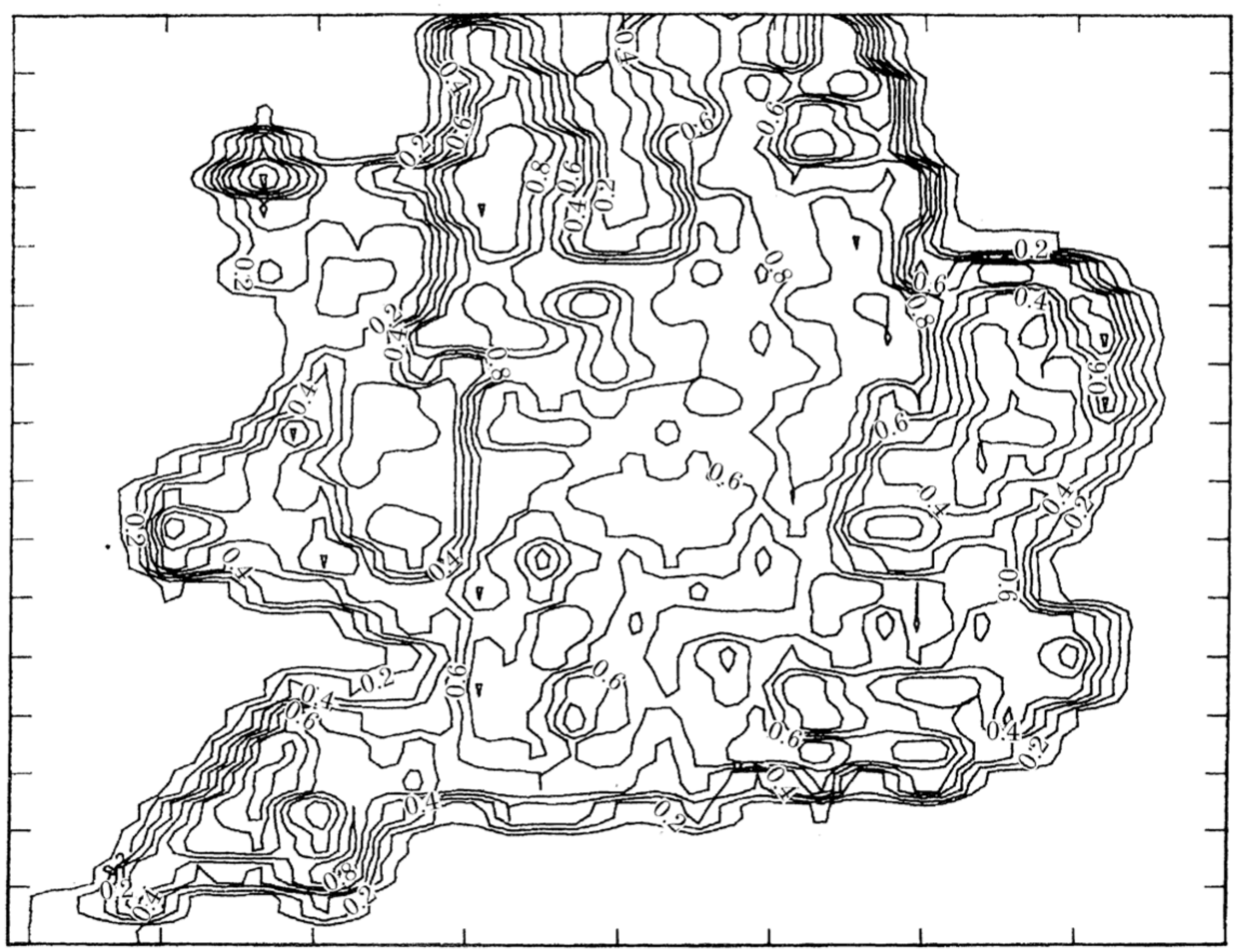

Math Biosciences (1991) **107**, 255-287.

DEPENDENCE OF EPIDEMIC AND POPULATION VELOCITIES ON BASIC PARAMETERS

Denis Mollison

As to the second wave, close inspection shows that the explanation lies, not so much in the determinism of the model, as in its modelling of the population as continuous rather than discrete and its associated inability to let population variables reach the value zero. Thus the density of infected at the place of origin of the epidemic never becomes zero, it only declines to a minimum of around one atto-fox (10^{-18} of a fox, Hughes 1960) per square kilometre. The model then allows this atto-fox to start the second wave as soon as the susceptible population has regrown sufficiently.

Equations de diffusion-reaction

parenthèse historique

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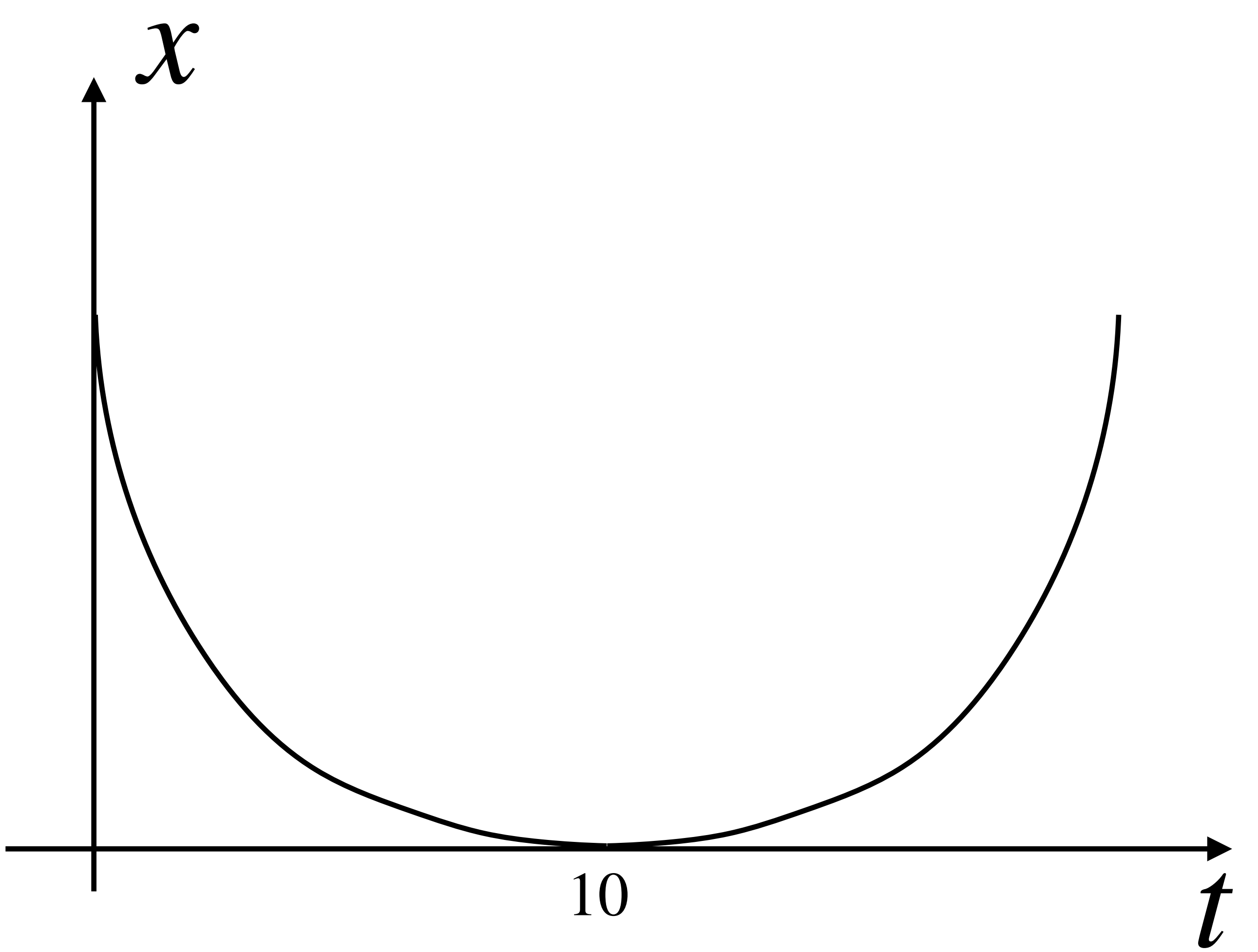
$$\frac{dx}{dt} = (-10 + t)x$$

$$x(t) = x(0)\mathrm{e}^{\left(-10t+\frac{t^2}{2}\right)}$$

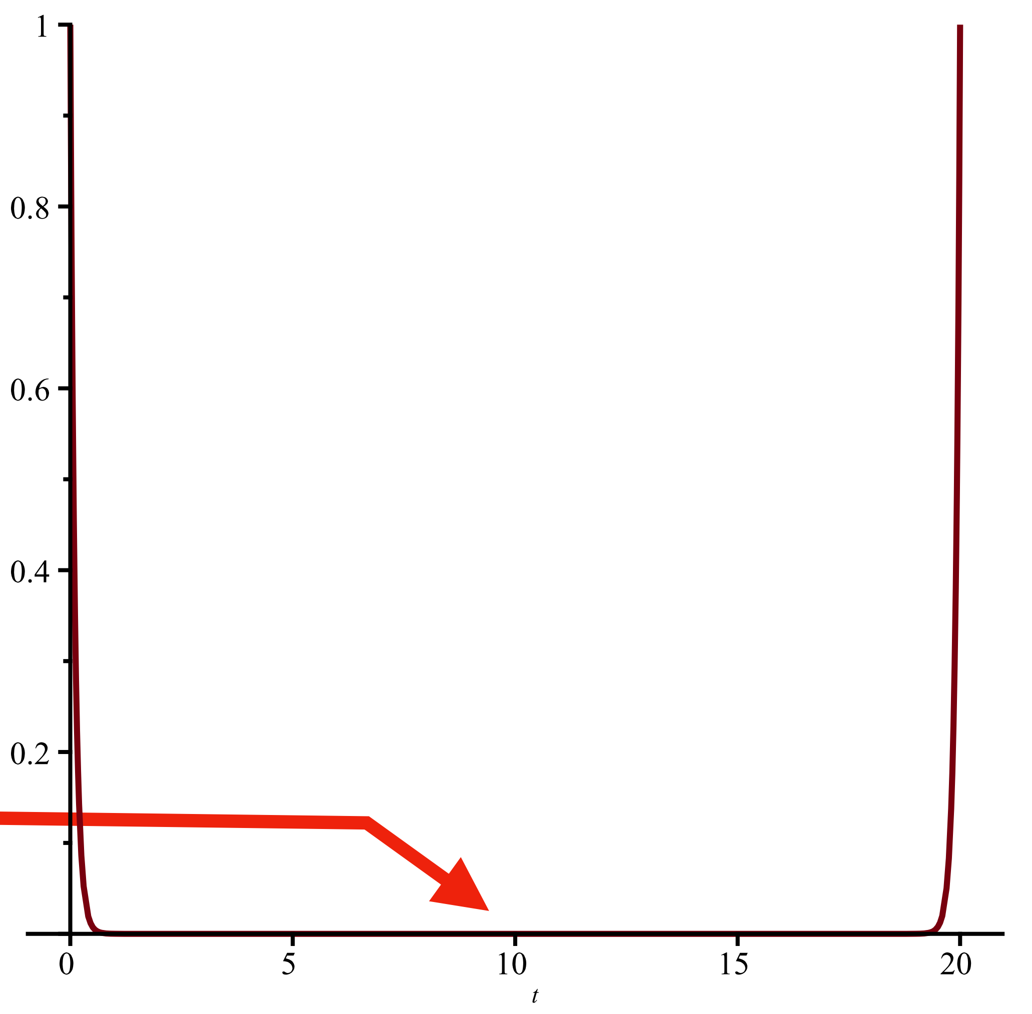


As to the second wave, close inspection shows that the explanation lies, not so much in the determinism of the model, as in its modelling of the population as continuous rather than discrete and its associated inability to let population variables reach the value zero. Thus the density of infected at the place of origin of the epidemic never becomes zero, it only declines to a minimum of around one atto-fox (10^{-18} of a fox, Hughes 1960) per square kilometre. The model then allows this atto-fox to start the second wave as soon as the susceptible population has regrown sufficiently.

$$\frac{dx}{dt} = (-10 + t)x$$

$$x(t) = x(0)\mathrm{e}^{\left(-10t+\frac{t^2}{2}\right)}$$

$$\min = x(0)1.92\,10^{-22}$$



Equations de diffusion-reaction

parenthèse historique

Bulletin of Mathematical Biology (2021) 83:104
https://doi.org/10.1007/s11538-021-00936-x

Society for Mathematical Biology

SPECIAL ISSUE: CELEBRATING J. D. MURRAY

https://people.maths.ox.ac.uk/fowler/

Atto-Foxes and Other Minutiae

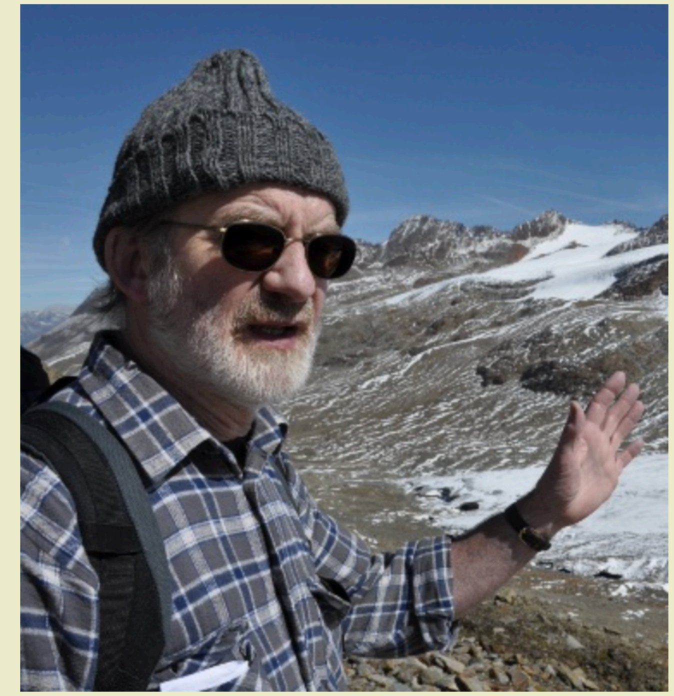
A. C. Fowler[1,2]

Received: 29 March 2020 / Accepted: 13 August 2021 / Published online: 31 August 2021

2 Atto-Foxes

An enduring issue in population dynamics is what has been called ‘the atto-fox problem’ (Lobry and Sari 2015). The origin of this term lies in a model suggested by

Andrew Fowler



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Equations de diffusion-reaction

parenthèse historique

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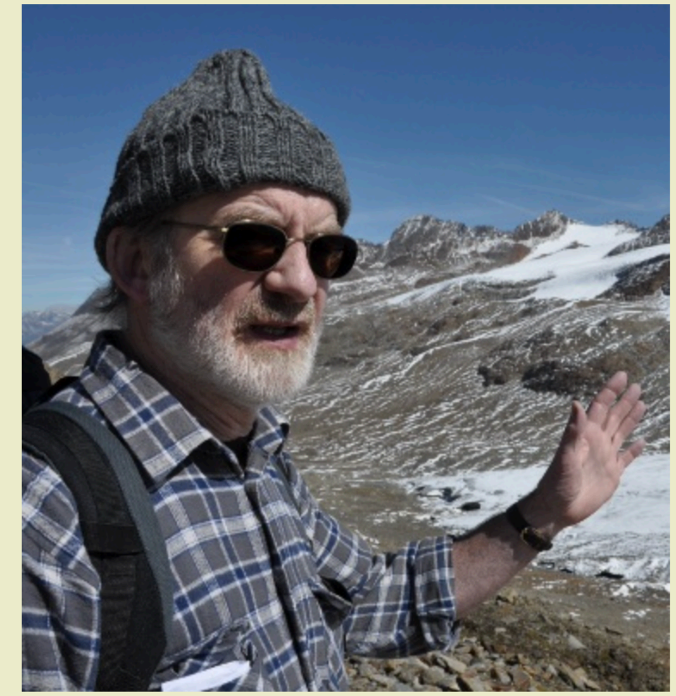

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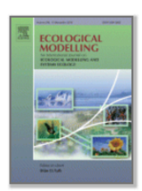
E-mail: fowler@maths.ox.ac.uk or andrew.fowler@ul.ie
University of Oxford phone: 44-1865-270519
27
8867

Lobry C, Sari T (2015) Migrations in the Rosenzweig-MacArthur model and the atto-fox problem. ARIMA J 20:95–125

Encore un peu de publicité

ELSEVIER

Ecological Modelling
Volume 246, 10 November 2012, Pages 1-10



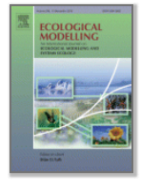
Effect of population size in a predator–prey model

F. Campillo, C. Lobry

Campillo-Lobry

parenthèse historique

Ecological Modelling
Volume 246, 10 November 2012, Pages 1-10

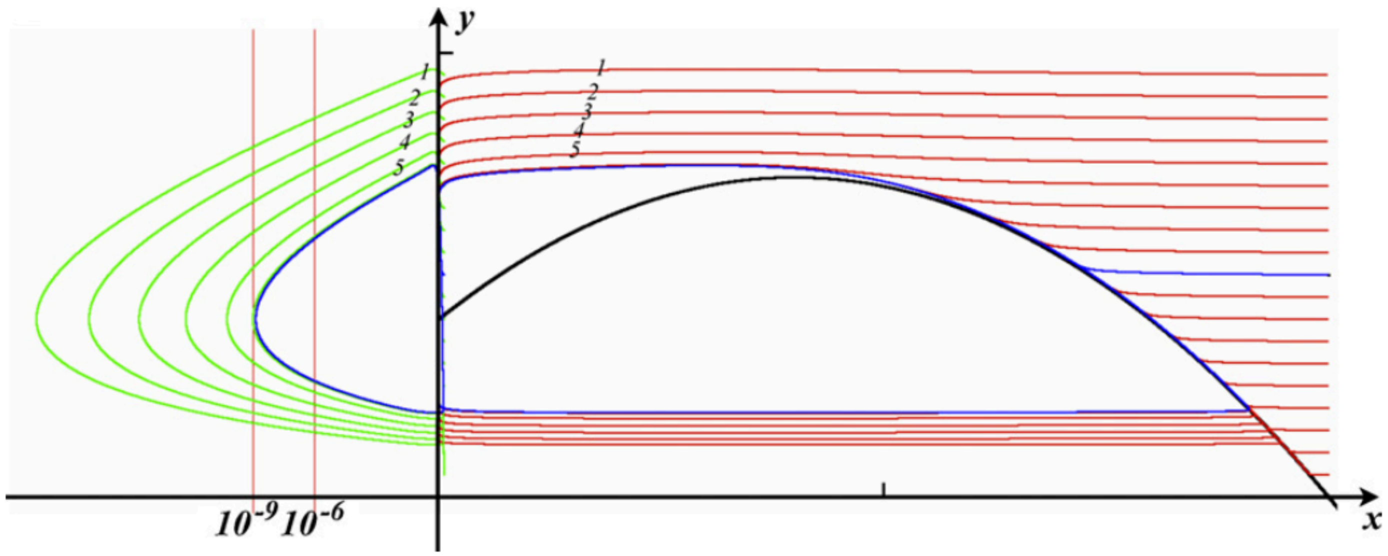


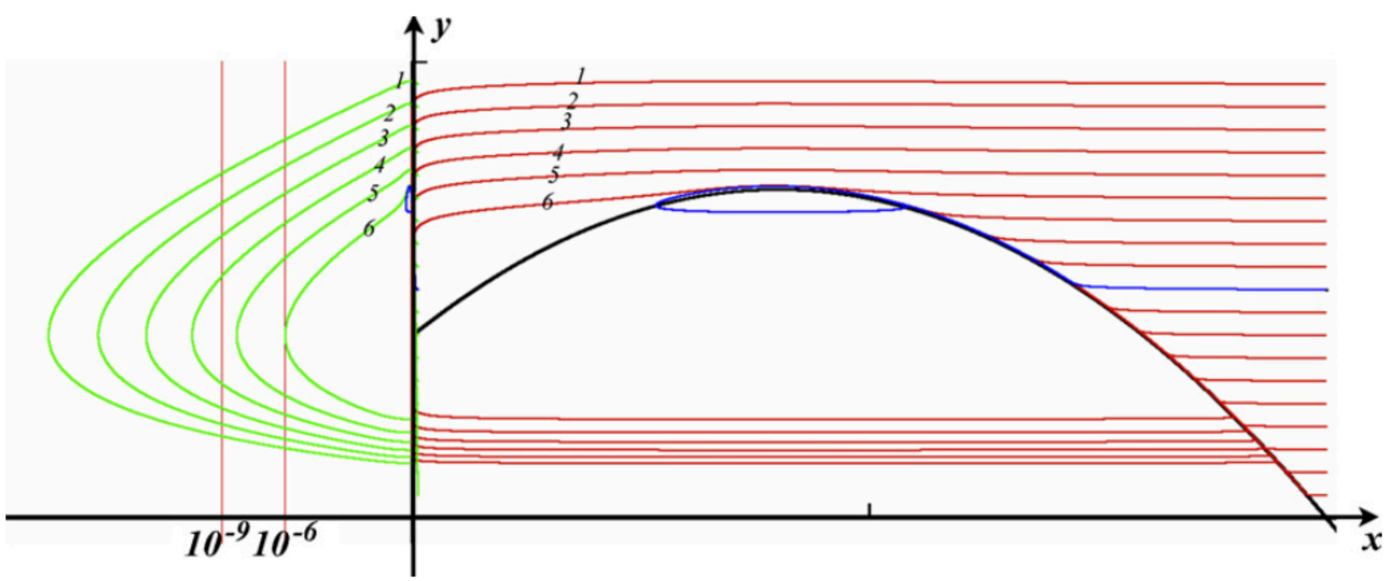
Effect of population size in a predator–prey model

$$\begin{cases} \dfrac{dx}{dt} = \dfrac{1}{\varepsilon}\left[f(x) - \mu(x)y\right], \\ \dfrac{dy}{dt} = (\mu(x) - m)y \end{cases}$$

$$f(x) = x(2-x)$$

$$\mu(x) = \frac{x}{0.4+x}, \ \varepsilon = 0.02$$



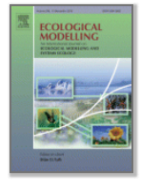


Théorie mathématique des ''Canards''
ANS - Reeb (1920-1993)

Albuquerque
Lutz
Harthong
Benoit
Callot
Diener F.
Diener M.
Sari……
………..

Reeb
Thom
Serre

Ecological Modelling
Volume 246, 10 November 2012, Pages 1-10

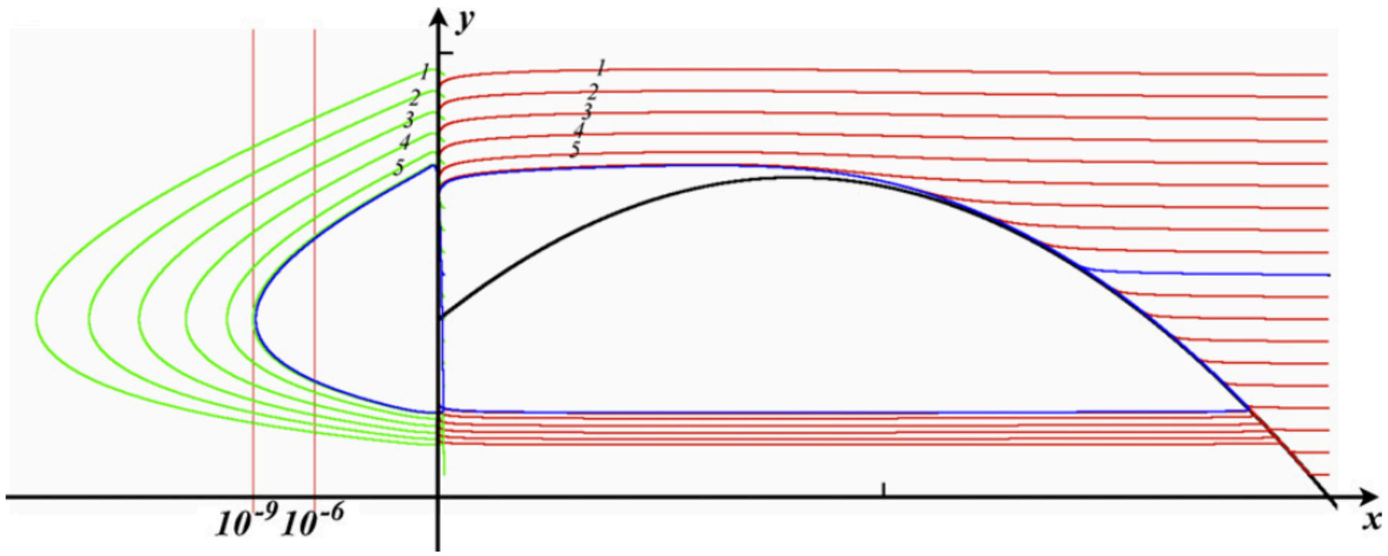


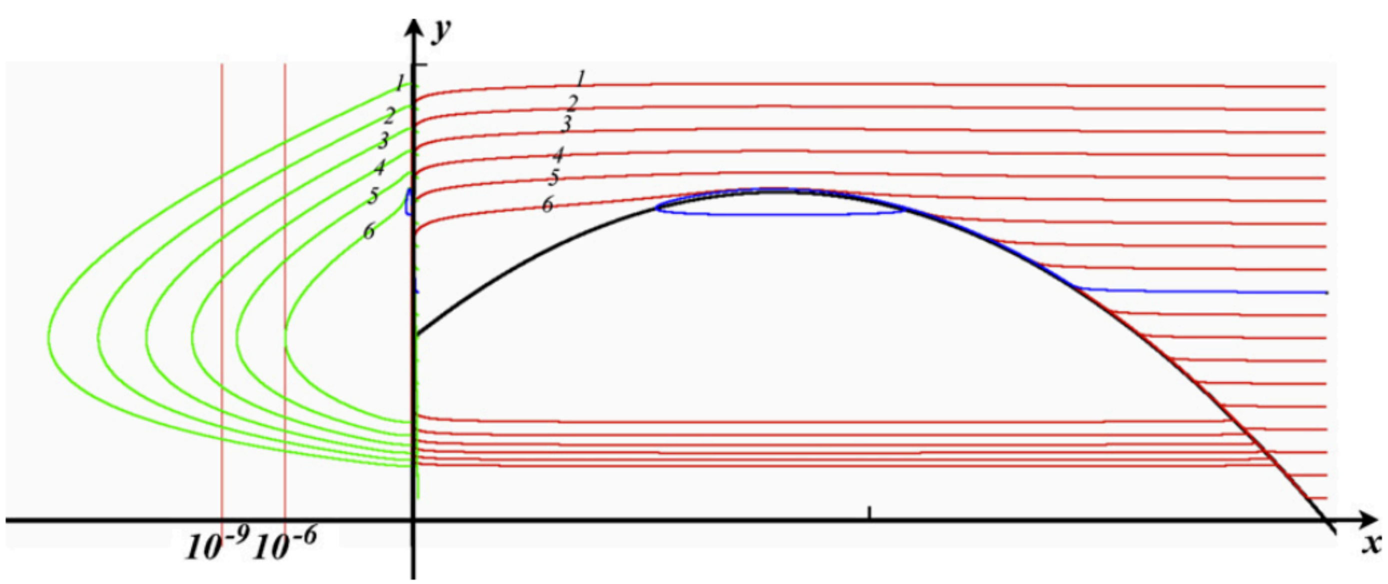
parenthèse historique

Effect of population size in a predator–prey model

$$\begin{cases} \dfrac{dx}{dt} = \dfrac{1}{\varepsilon}\left[f(x) - \mu(x)y\right], \\[2mm] \dfrac{dy}{dt} = (\mu(x) - m)y \end{cases} \qquad f(x) = x(2-x)$$

$$\mu(x) = \frac{x}{0.4+x}, \ \varepsilon = 0.02$$





Bruit démographique

$$\begin{cases} \dfrac{dx}{dt} = f(x) - \mu(x)y \ + \sigma_1(\omega, \ldots)\dfrac{dB_1}{dt} \\[2mm] \dfrac{dy}{dt} = (c\,\mu(x) - \delta)y \ + \sigma_2(\omega, \ldots)\dfrac{dB_2}{dt} \end{cases}$$

$\omega = 10^n =$ # **d'individus/unité**

Ecological Modelling
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parenthèse historique

Effect of population size in a predator–prey model

$$\begin{cases} \dfrac{dx}{dt} = \dfrac{1}{\varepsilon}\left[f(x) - \mu(x)y\right], \\ \dfrac{dy}{dt} = (\mu(x) - m)y \end{cases}$$

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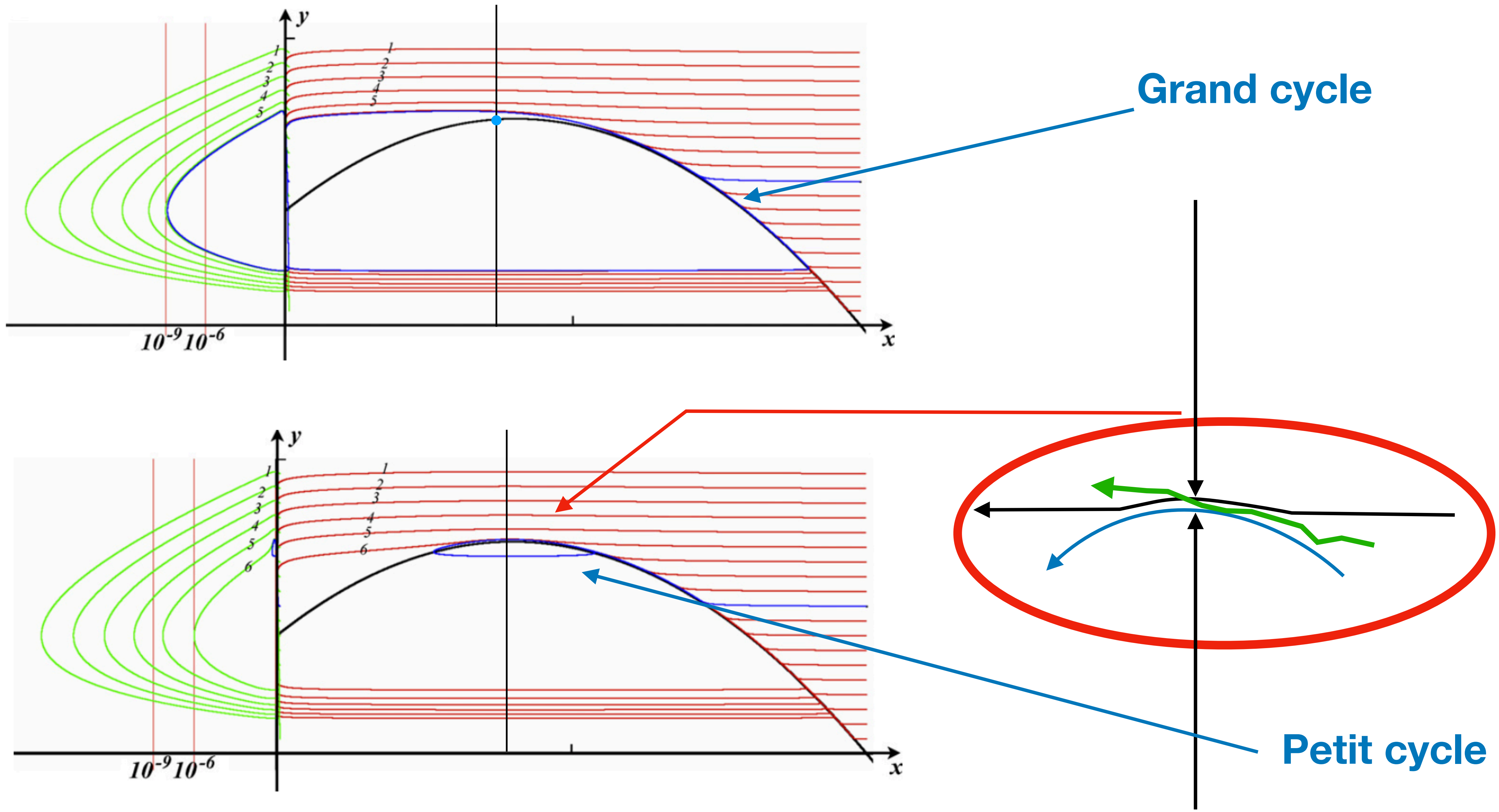

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parenthèse historique

F. Campillo, C. Lobry / Ecolog

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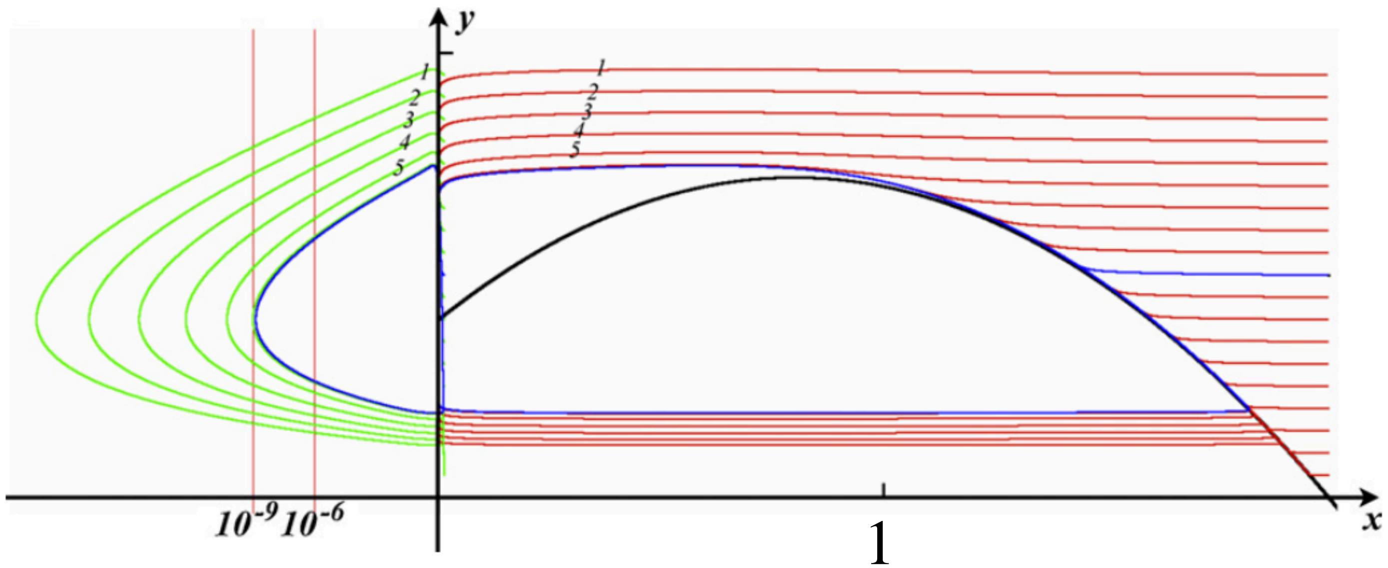

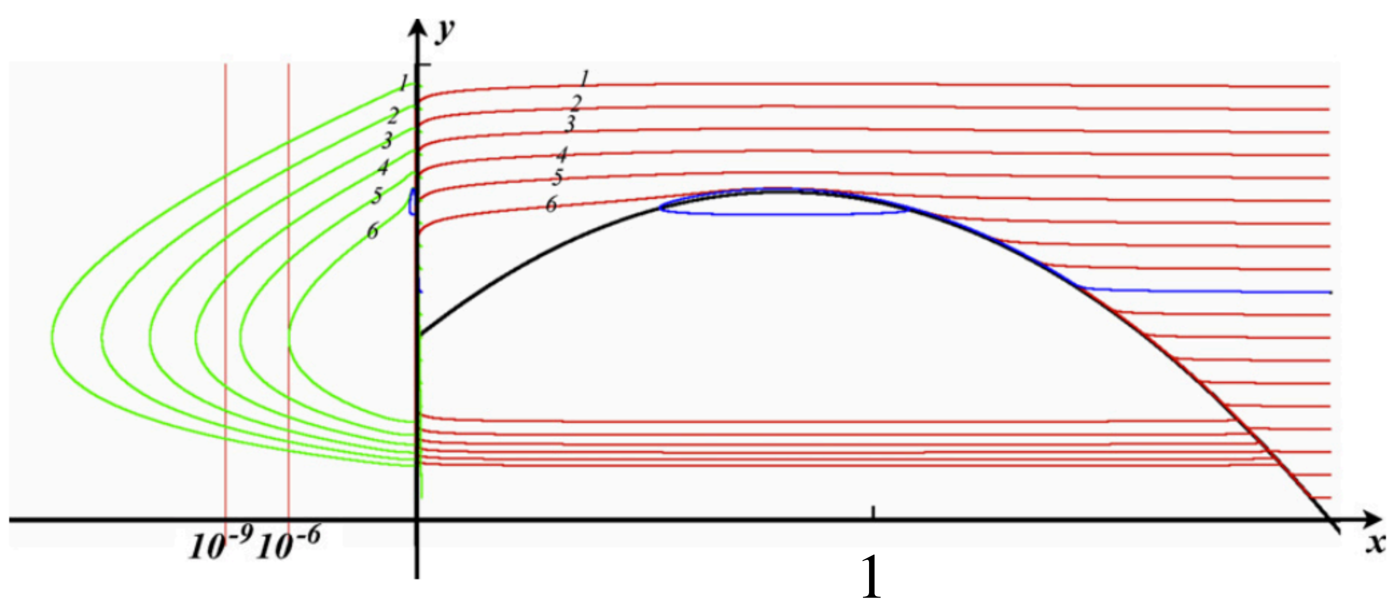

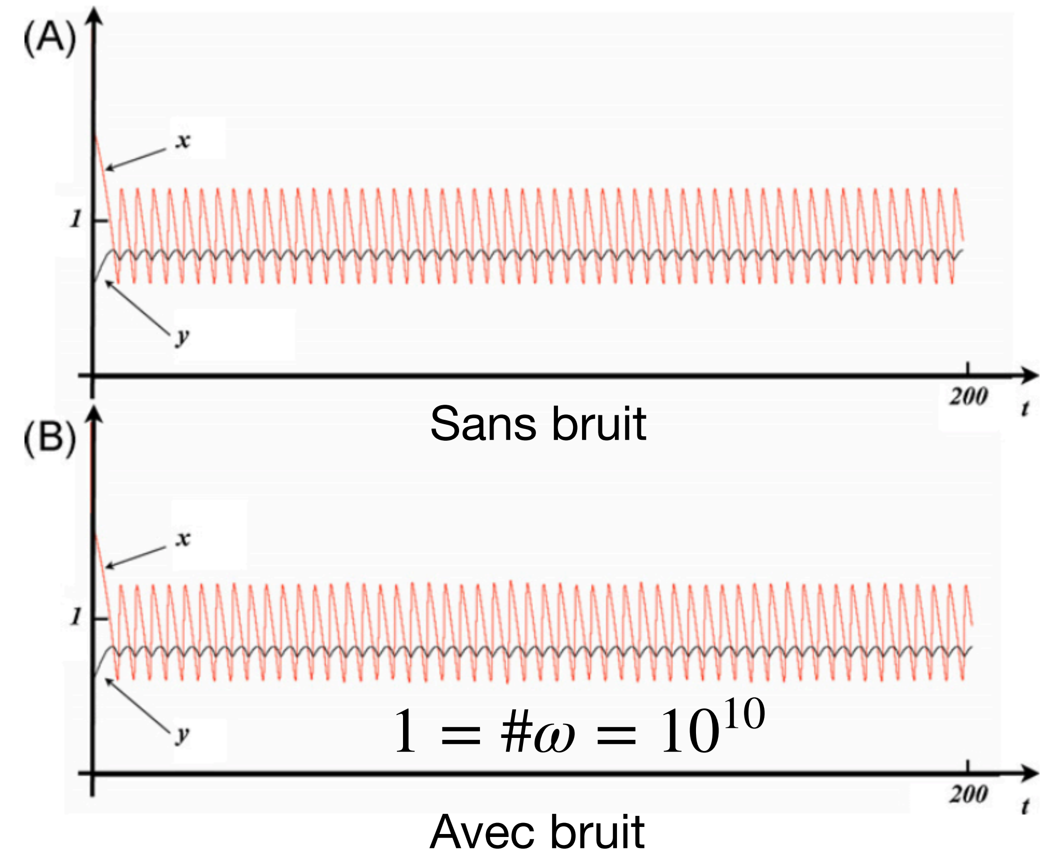

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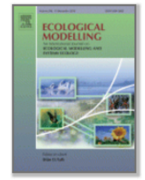

parenthèse historique

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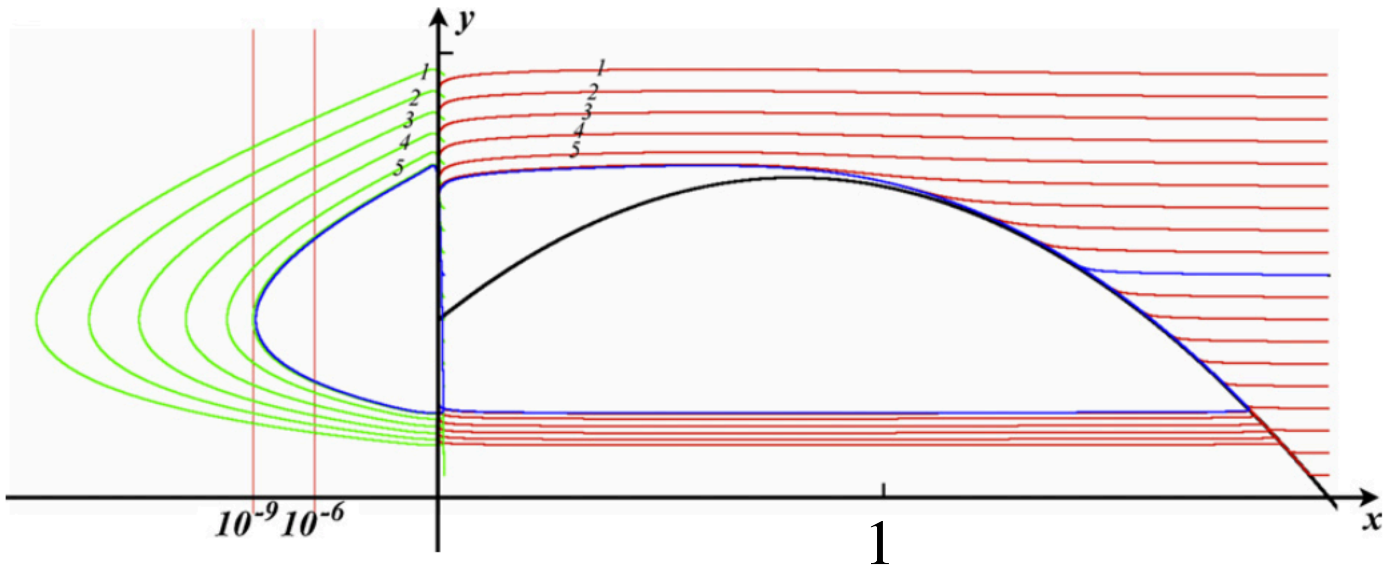
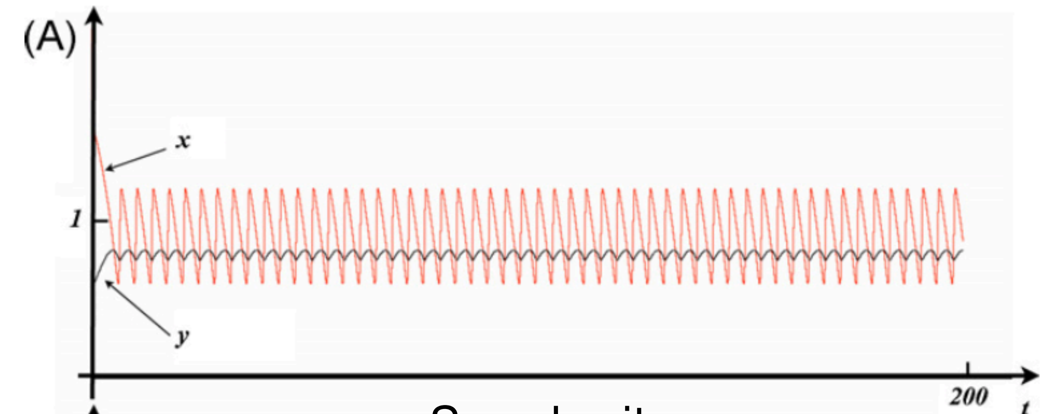
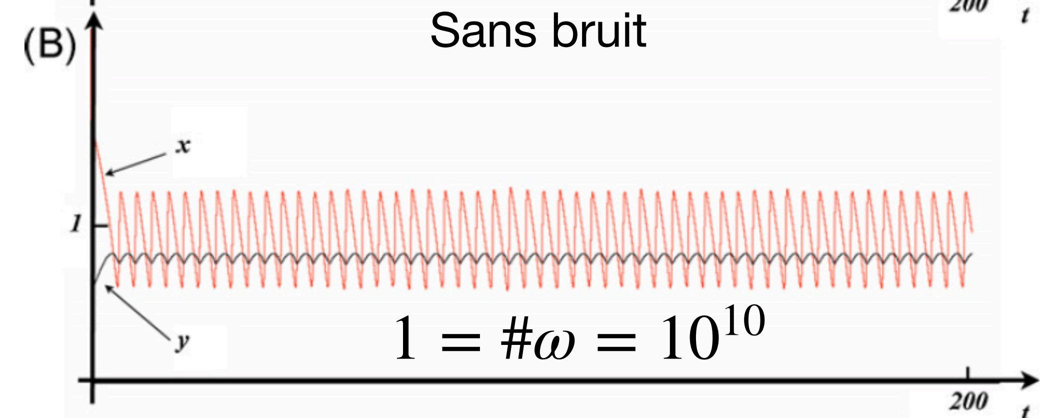
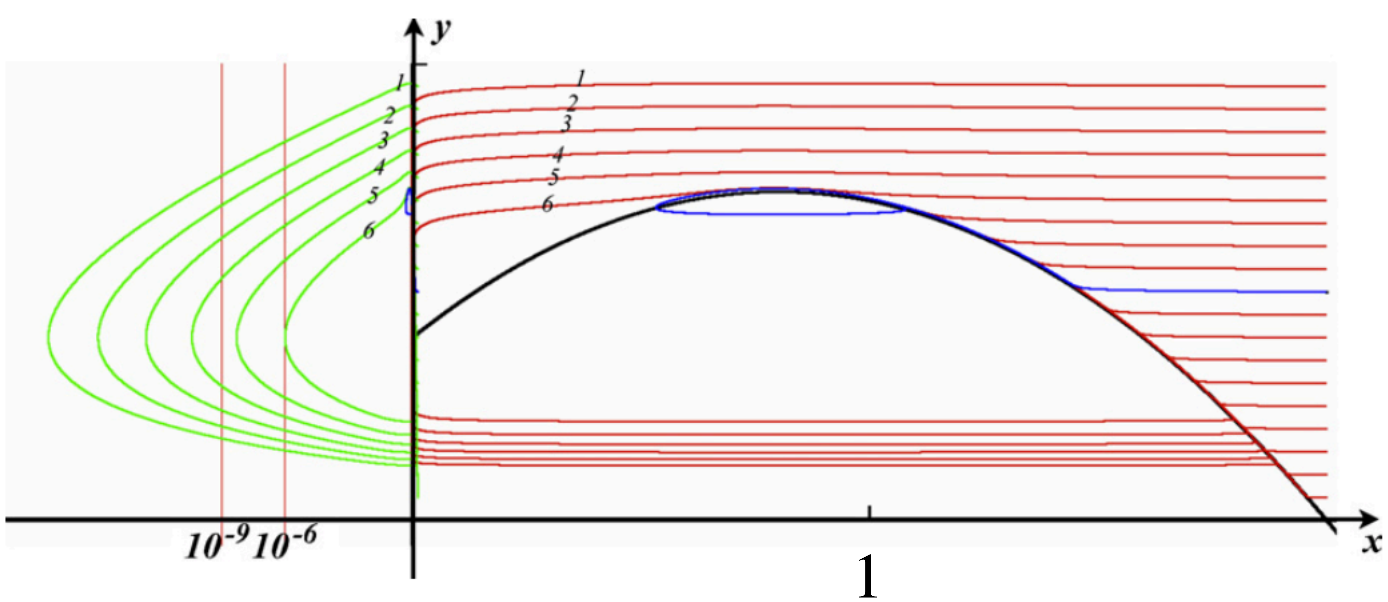


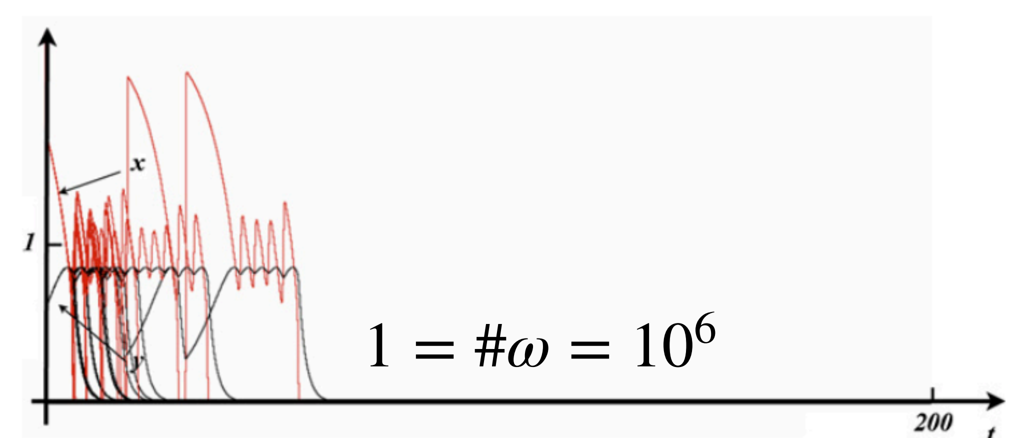


Sans bruit



Avec bruit





Avec bruit

Conclusion de l’introduction

On s’intéresse à deux sites (seulement)

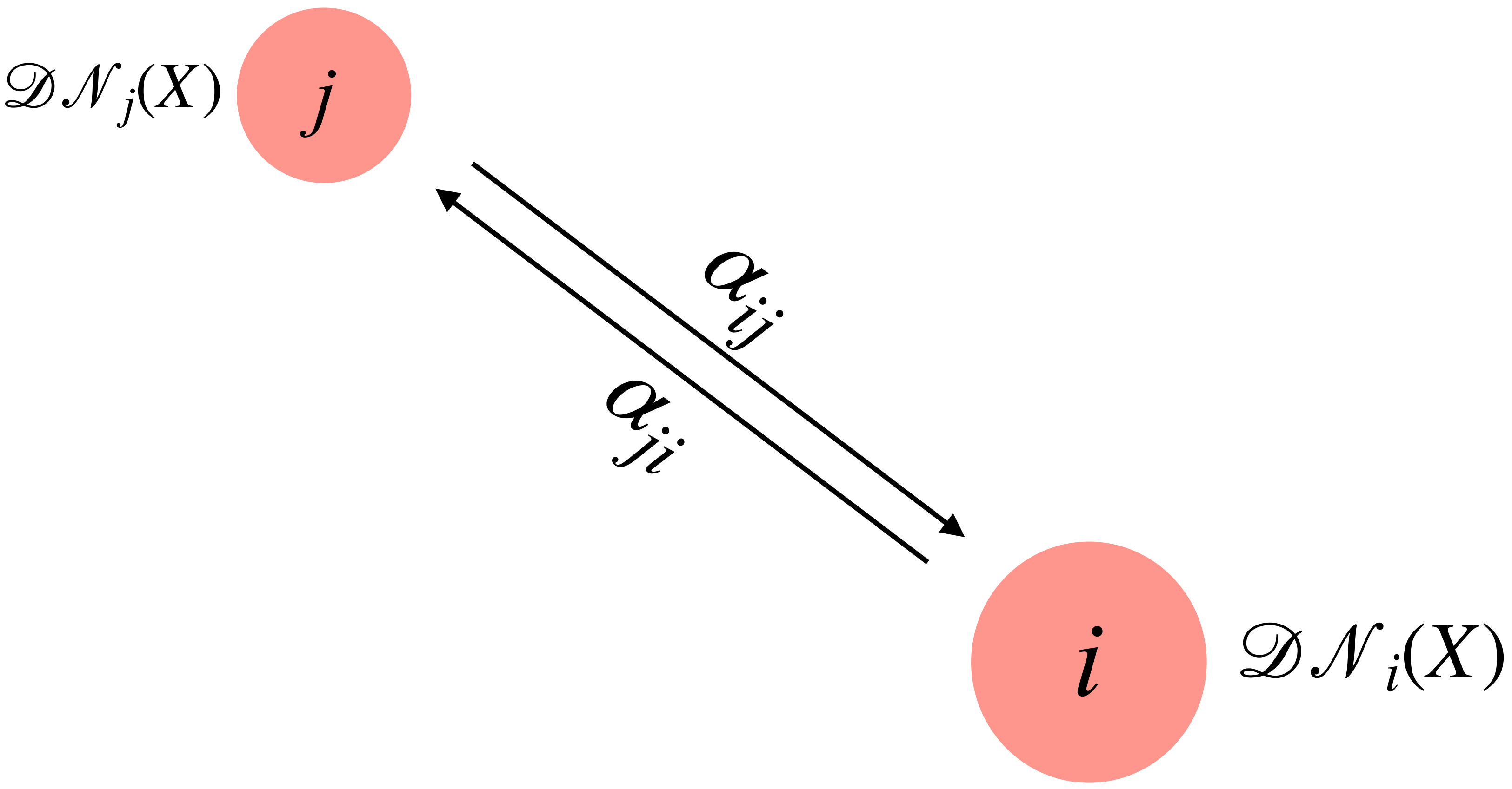
On cherche à comprendre l’effet de la migration sur la croissance

(ou la persistance)

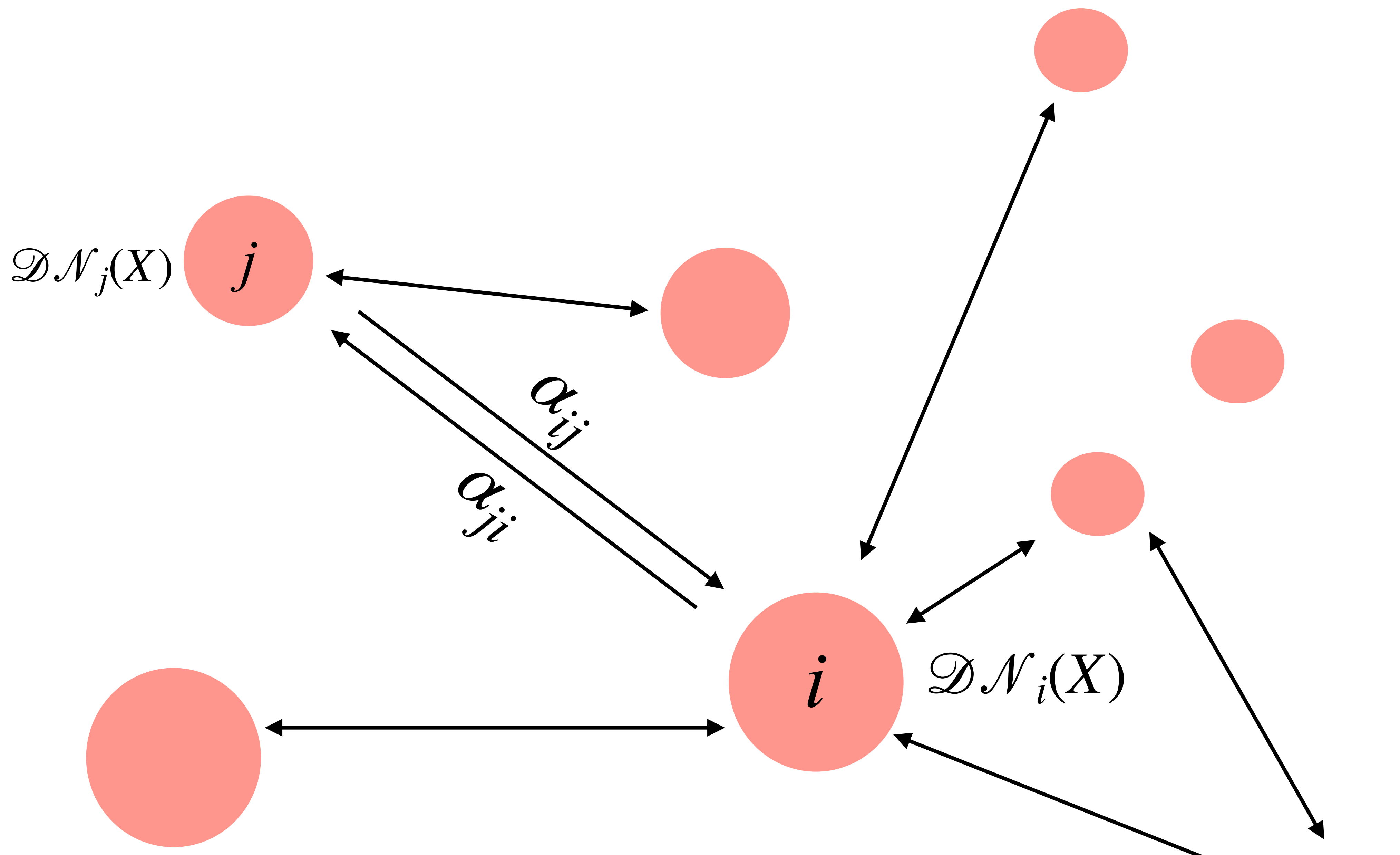
Sans oublier l’atto fox

Avec la propagation des épidémies en perspective

Modèles Multi-sites



Modèles Multi-sites



$$\frac{dx_i}{dt} = f(t, x_i) + \sum_{ij} \alpha_{ij} x_j$$

Modèles Multi-sites

Source

Source

La population croit
La population est persistante

$$\frac{dx}{dt} = r\,x \qquad \frac{dx}{dt} = r\,x\left(1 - \frac{x}{K}\right)$$

Puits

Sink

La population décroit
La population disparait

Proc. Natl. Acad. Sci. USA
Vol. 95, pp. 3696–3698, March 1998
Ecology

Populations can persist in an environment consisting of sink habitats only

VINCENT A. A. JANSEN*†‡ AND JIN YOSHIMURA*§¶

*NERC Centre for Population Biology, Imperial College at Silwood Park, Ascot, Berkshire SL5 7PY, United Kingdom; §Department of Systems Engineering, Shizuoka University, 3-5-1 Johoku, Hamamatsu, 432 Japan; and ¶Department of Environmental and Forest Biology, State University of New York, College of Environmental Science and Forestry, Syracuse, NY 13210

Modèles Deux-sites

Bien comprendre le cas de 2-sites
pour
pouvoir aller plus loin

Migrations in the Rosenzweig-MacArthur model and the "atto-fox" problem

Claude Lobry* — Tewfik Sari **

ARIMA Journal, vol. 20, pp. 95-125 (2015)

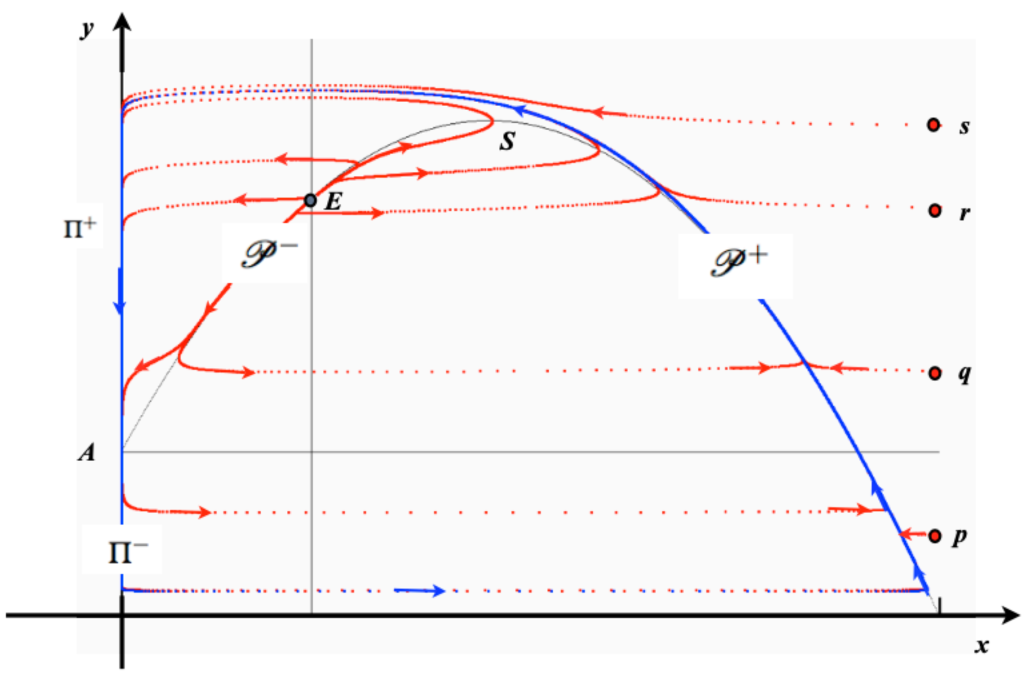
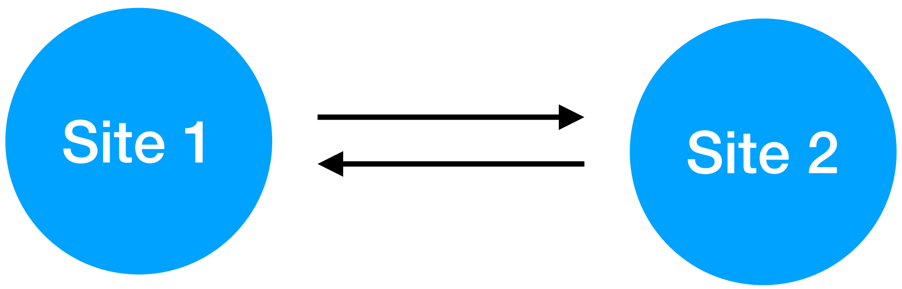


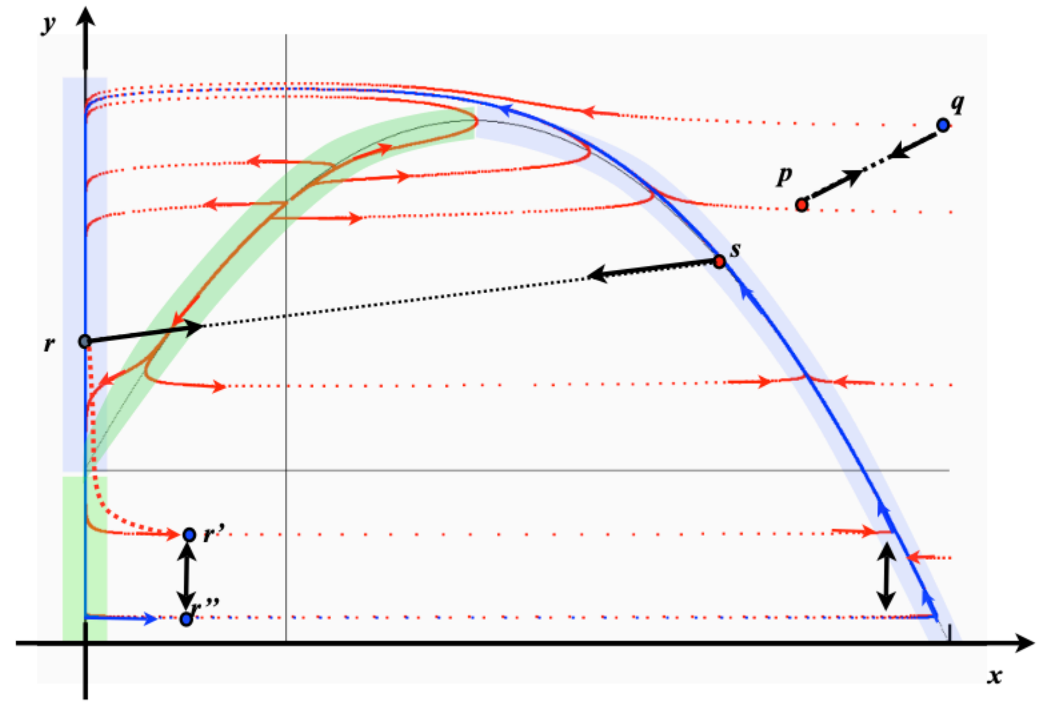
Figure 2. *Phase portrait of (3) :* $\varepsilon = 0.05$, $m : 0.7$

$$\begin{cases} \dfrac{dx}{dt} = \dfrac{1}{\varepsilon}\left(2x(1-x) - \dfrac{x}{0.1+x}y\right) \\ \dfrac{dy}{dt} = \left(\dfrac{x}{0.1+x} - m\right)y \end{cases}$$

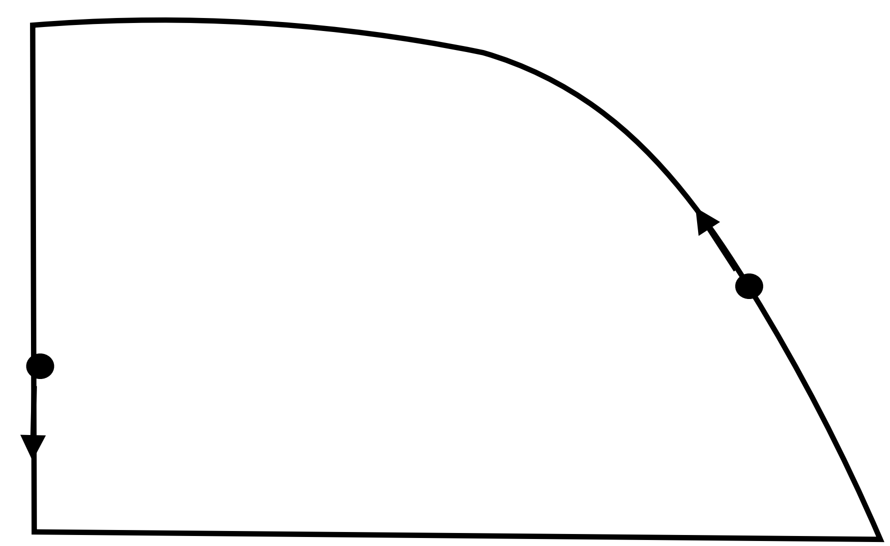
y_0	0.4	0.5	0.6	0.7
$\exp(\frac{\xi_{min}}{\varepsilon})$	$1.66 \quad 10^{-9}$	$1.79 \quad 10^{-16}$	$2.14 \quad 10^{-24}$	$5.34 \quad 10^{-33}$
x_{min}	$1.62 \quad 10^{-9}$	$1.69 \quad 10^{-16}$	$1.90 \quad 10^{-24}$	$4.31 \quad 10^{-33}$

Values of the parameters : $\varepsilon = 0.05$; $m = 0.7$; $x_0 = 0.05$; $\xi_0 = \varepsilon \log(x_0)$



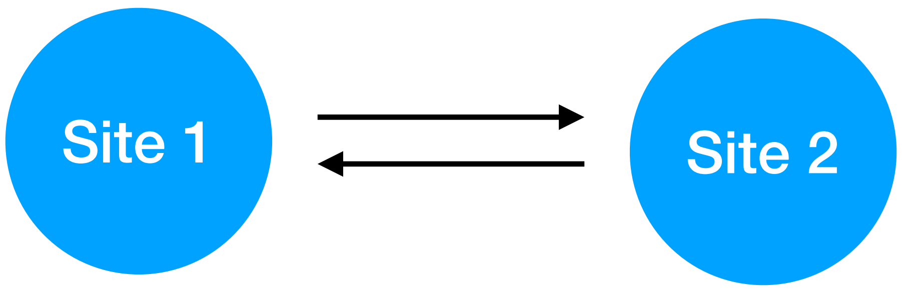


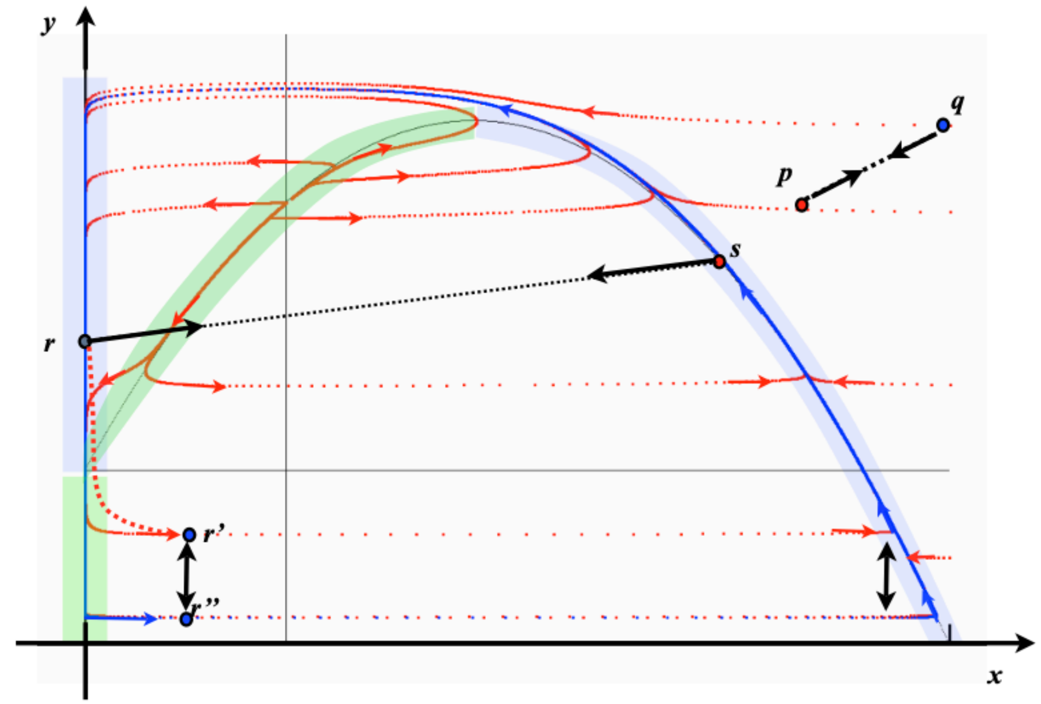
$$
\left\{
\begin{aligned}
\frac{dx_1}{dt} &= \frac{1}{\varepsilon}(x_1 g(x_1) - h(x_1)y_1) + k(x_2 - x_1)\\
\frac{dy_1}{dt} &= (h(x_1) - m)y_1 + k(y_2 - y_1)\\
\frac{dx_2}{dt} &= \frac{1}{\varepsilon}(x_2 g(x_2) - h(x_2)y_2) + k(x_1 - x_2)\\
\frac{dy_2}{dt} &= (h(x_2) - m)y_2 + k(y_1 - y_2)
\end{aligned}
\right.
$$



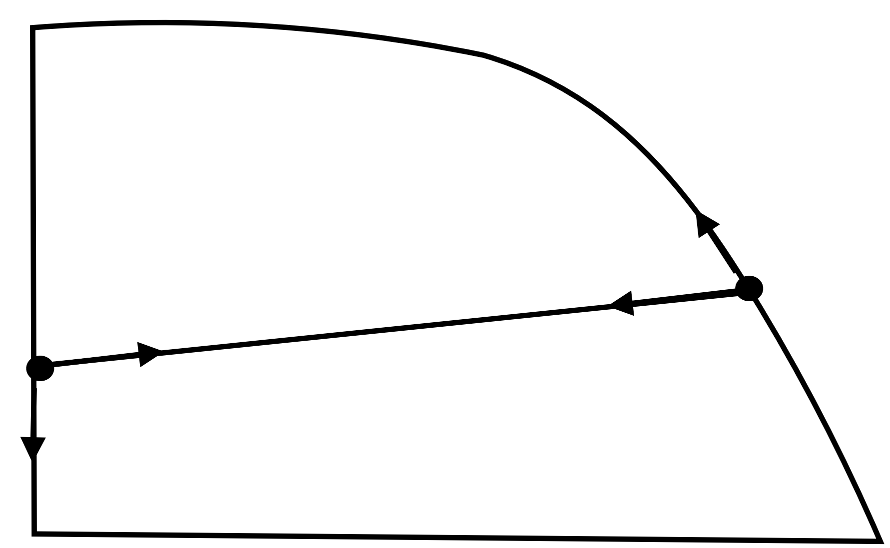
Modèles Deux-sites

Et Atto-Fox



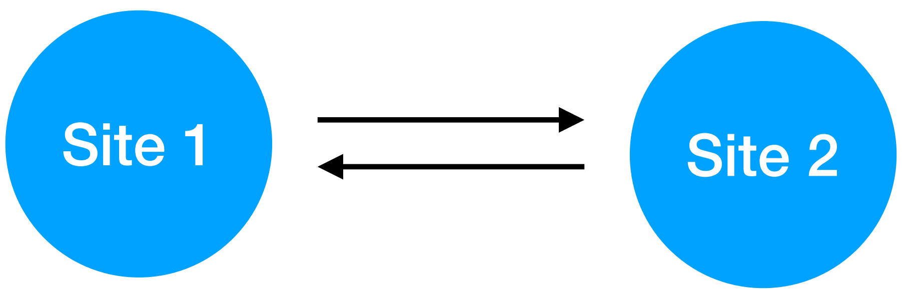


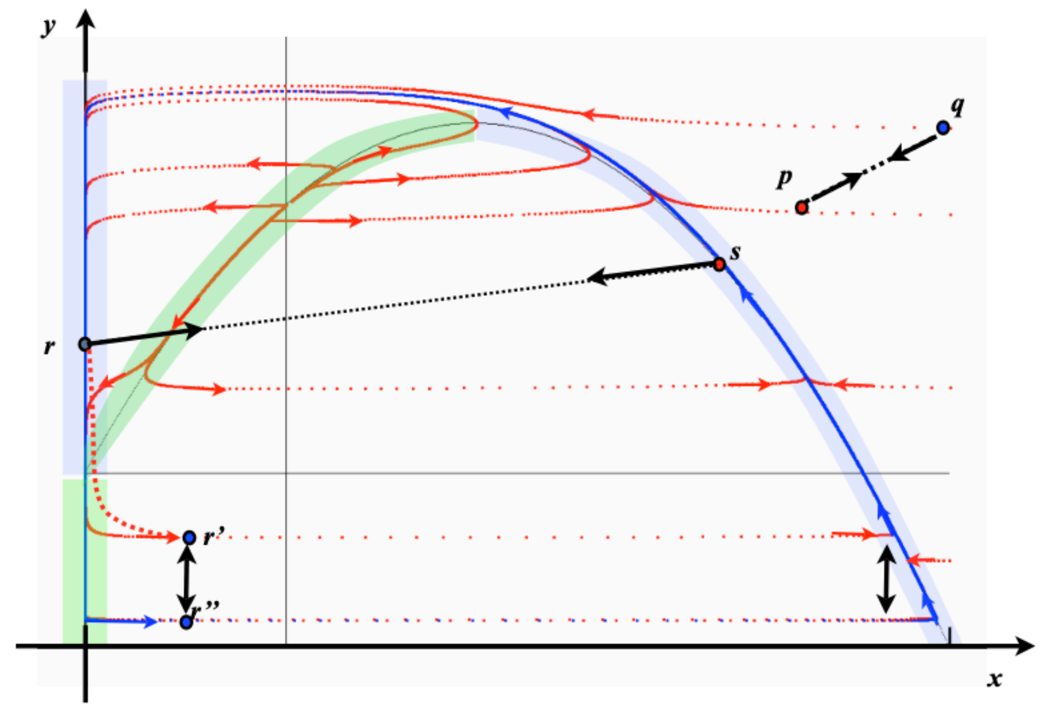
$$\left\{\begin{array}{rcl}
\dfrac{dx_1}{dt} & = & \dfrac{1}{\varepsilon}(x_1 g(x_1) - h(x_1)y_1) + k(x_2 - x_1) \\
\dfrac{dy_1}{dt} & = & (h(x_1) - m)y_1 + k(y_2 - y_1) \\
\dfrac{dx_2}{dt} & = & \dfrac{1}{\varepsilon}(x_2 g(x_2) - h(x_2)y_2) + k(x_1 - x_2) \\
\dfrac{dy_2}{dt} & = & (h(x_2) - m)y_2 + k(y_1 - y_2)
\end{array}\right.$$



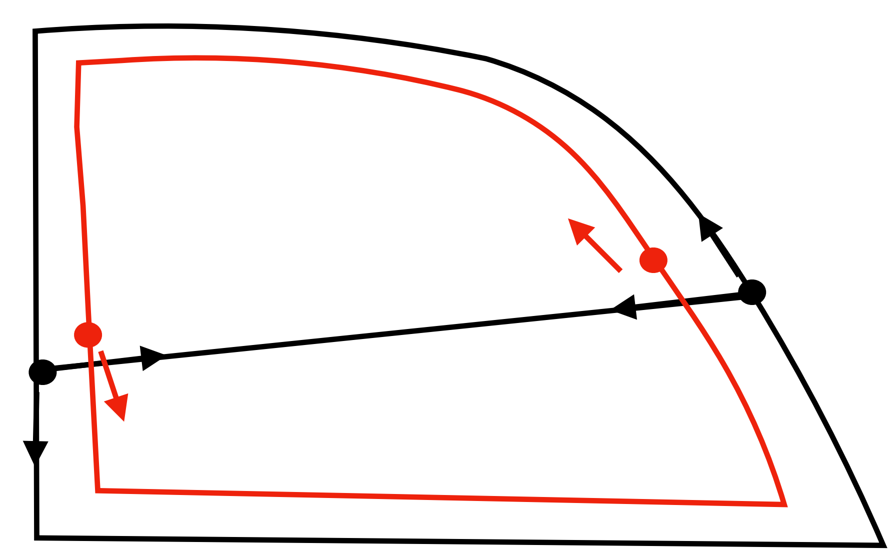
Modèles Deux-sites

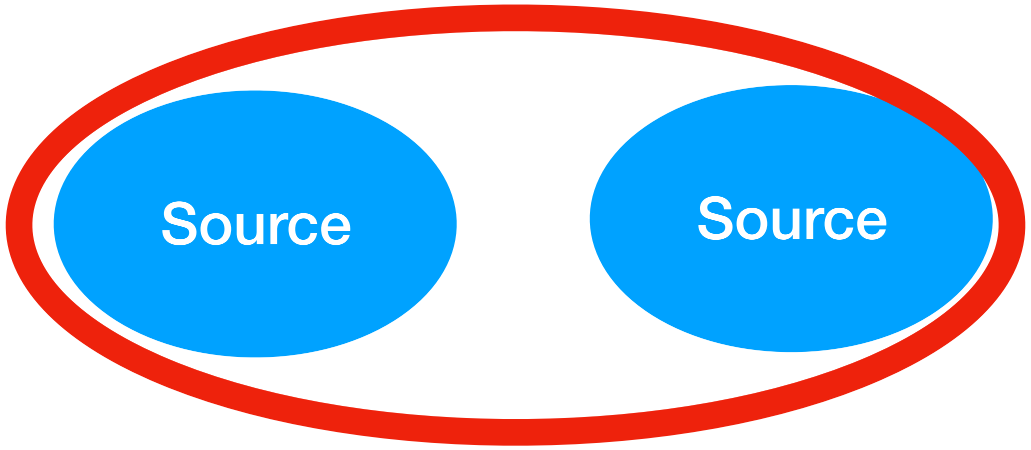
Et Atto-Fox

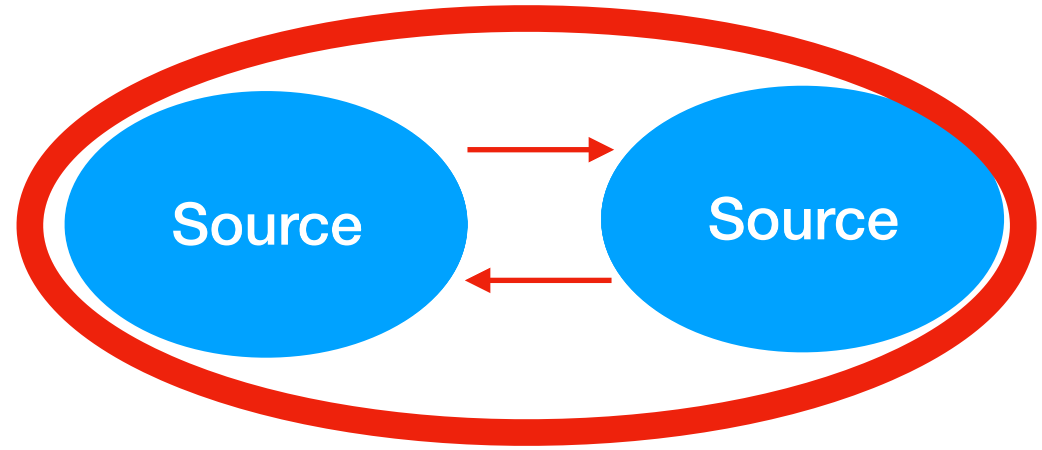




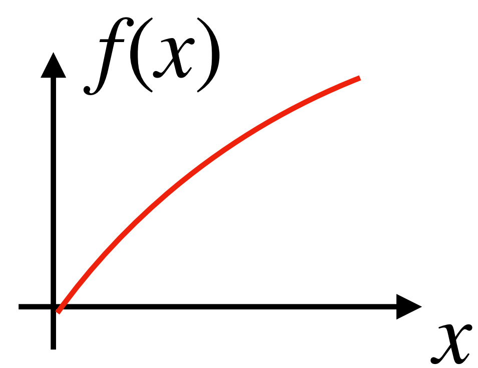
$$
\left\{
\begin{array}{rcl}
\dfrac{dx_1}{dt} & = & \dfrac{1}{\varepsilon}(x_1 g(x_1) - h(x_1)y_1) + k(x_2 - x_1) \\
\dfrac{dy_1}{dt} & = & (h(x_1) - m)y_1 + k(y_2 - y_1) \\
\dfrac{dx_2}{dt} & = & \dfrac{1}{\varepsilon}(x_2 g(x_2) - h(x_2)y_2) + k(x_1 - x_2) \\
\dfrac{dy_2}{dt} & = & (h(x_2) - m)y_2 + k(y_1 - y_2)
\end{array}
\right.
$$



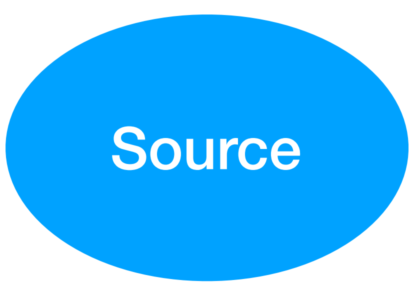




Que comparer ?

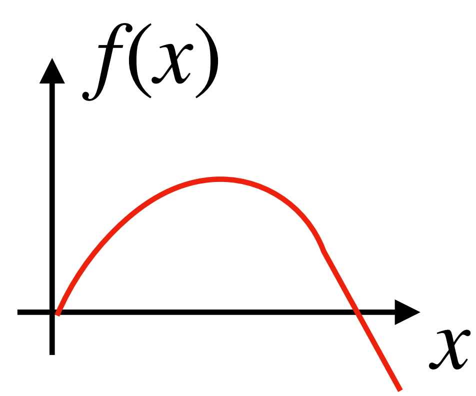


$$\frac{dx}{dt} = rx$$

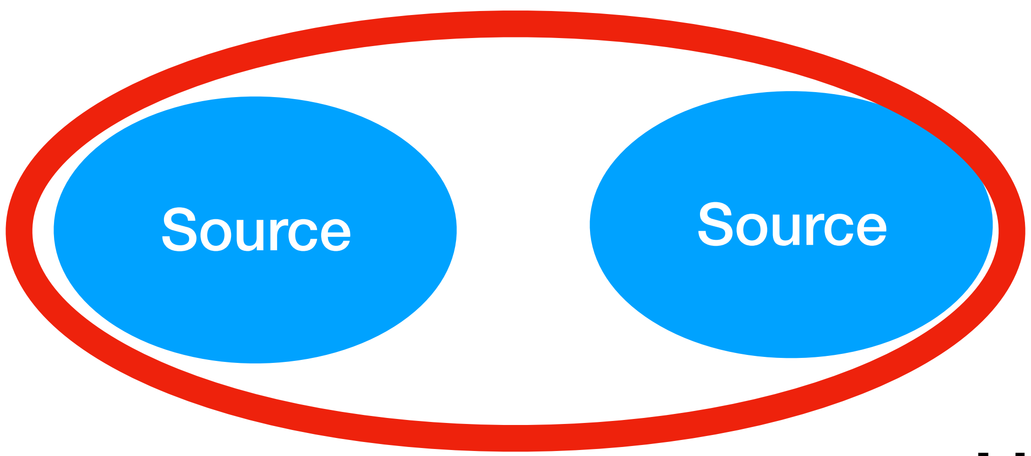


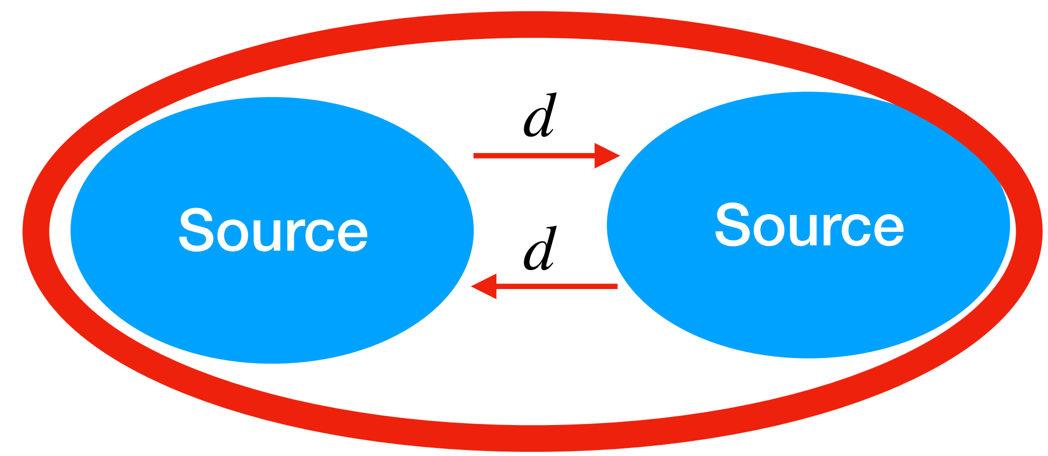
Dynamique sur 1 source ?

$$\frac{dx}{dt} = f(x)$$



$$\frac{dx}{dt} = r\left(x - \frac{x}{K}\right)$$



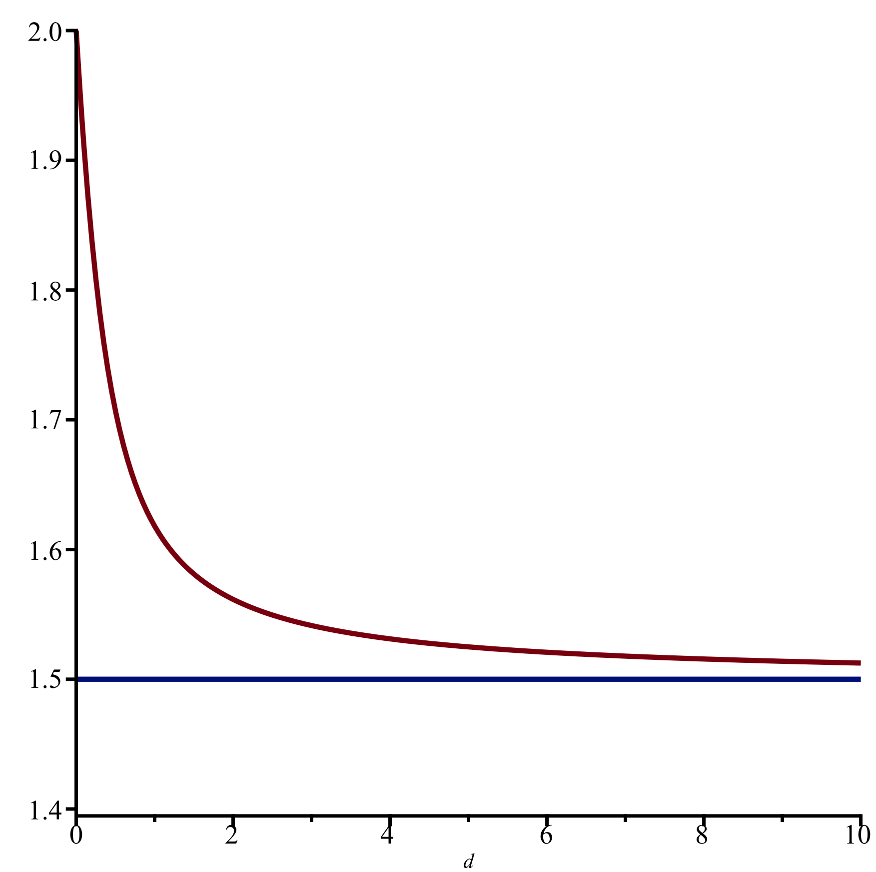


Linéaire

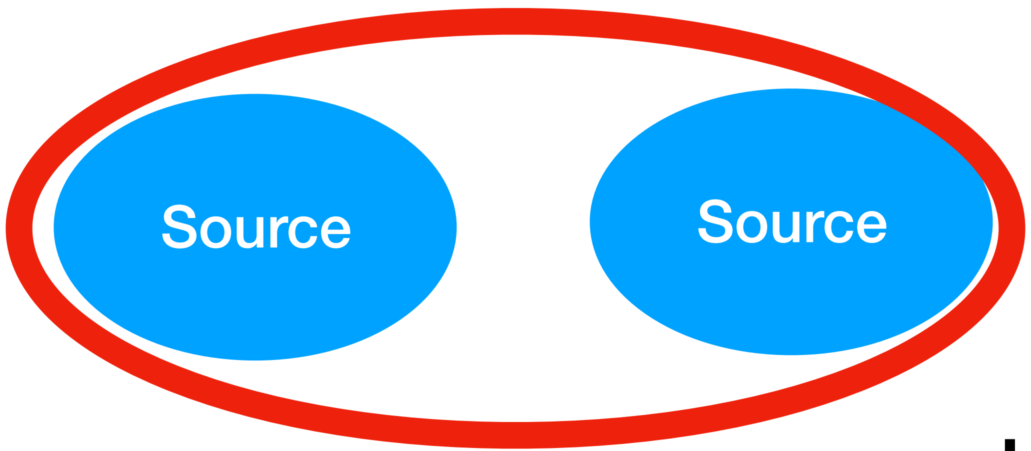
Comparer les taux de croissance

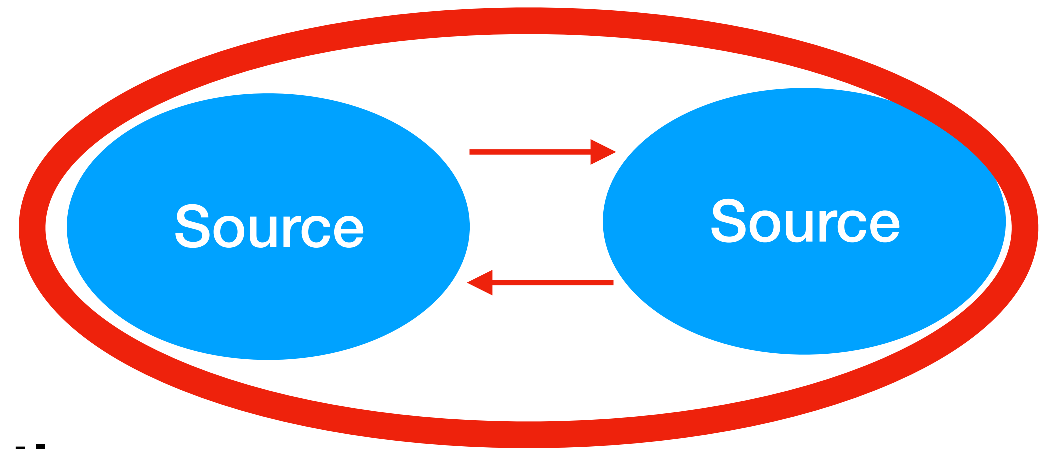
$$\frac{r1}{2} + \frac{r2}{2} - d + \frac{\sqrt{4\,d^2 + r1^2 - 2\,r1\,r2 + r2^2}}{2}$$

$$\begin{aligned} \frac{dx_1}{dt} &= r_1 x_1 + d(x_2 - x_1) \\ \frac{dx_2}{dt} &= r_2 x_2 + d(x_1 - x_2) \end{aligned}$$



En fait c'est pareil si les deux taux de croissance ne sont pas de même signe : (Source-Puits)





(Total Population Equilibrium)

Logistique
Comparer les"TPE"

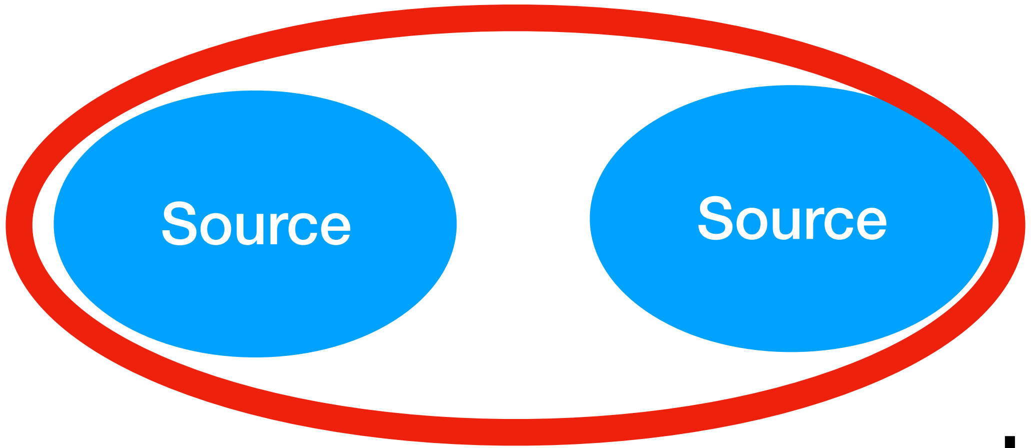
$$\frac{dx}{dt} = r\,x\left(1 - \frac{x}{K}\right) = \frac{r}{K}x(K - x) = \rho x(S - x)$$

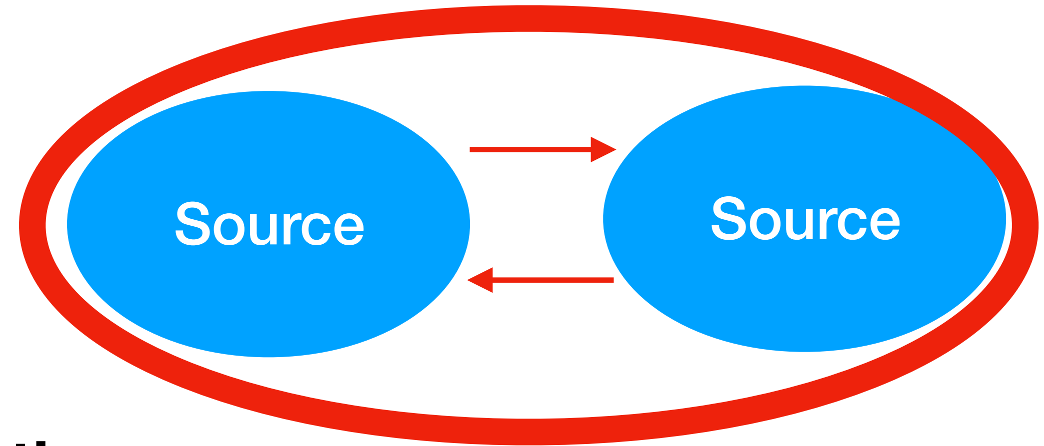
$$\begin{aligned} \frac{dx_1}{dt} &= rx_1\left(1 - \frac{x_1}{K_1}\right) + d(x_2 - x_1) \\ \frac{dx_2}{dt} &= rx_2\left(1 - \frac{x_2}{K_2}\right) + d(x_1 - x_2) \end{aligned}$$

$$\begin{aligned} 0 &= rx_1^*\left(1 - \frac{x_1^*}{K_1}\right) + d(x_2^* - x_1^*) \\ 0 &= rx_2^*\left(1 - \frac{x_2^*}{K_2}\right) + d(x_1^* - x_2^*) \end{aligned}$$

$$d = 0 \Longrightarrow K_1 + K_2$$

$$d => 0 \Longrightarrow x_1^* + x_2^*$$





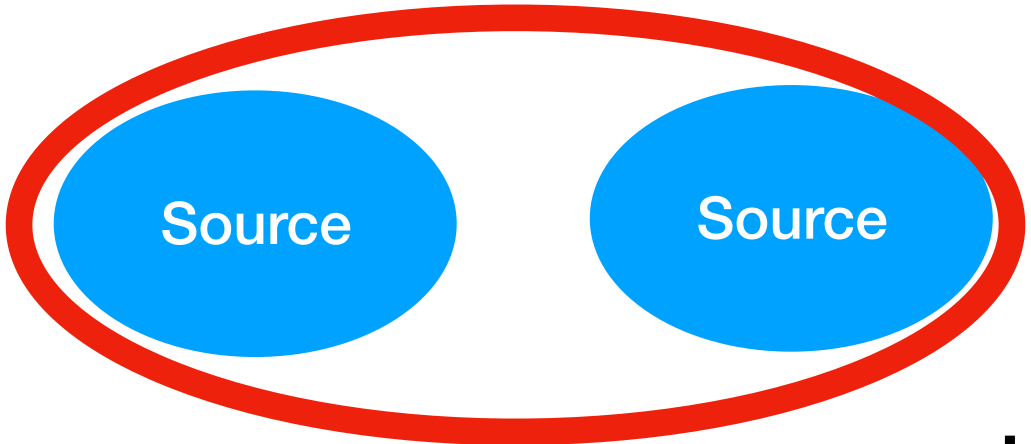
(Total Population Equilibrium)

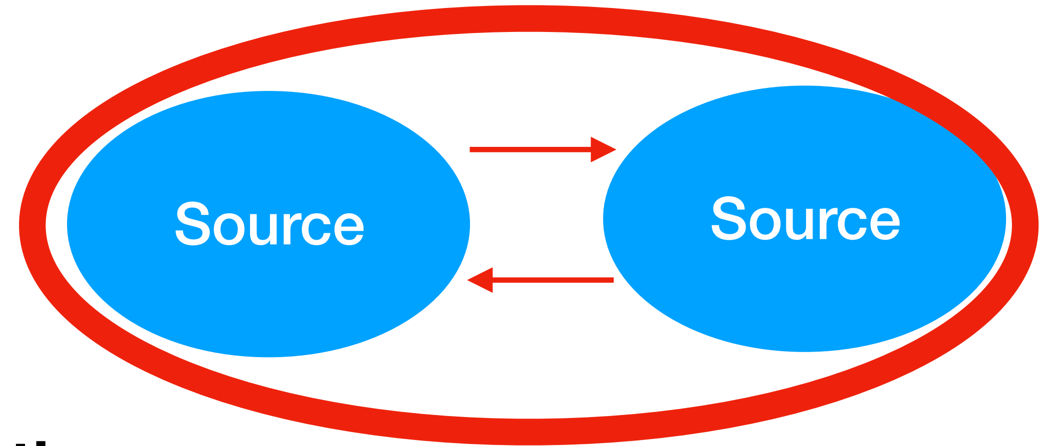
Logistique
Comparer les"TPE"

$$\frac{dx}{dt} = r\,x\left(1 - \frac{x}{K}\right) = \frac{r}{K}x(K - x) = \rho x(S - x)$$

$$\begin{aligned}
\frac{dx_1}{dt} &= \rho x_1(S_1 - x_1) + d(x_2 - x_1) \\
\frac{dx_2}{dt} &= \rho x_2(S_2 - x_2) + d(x_1 - x_2)
\end{aligned}$$

$$\begin{aligned}
0 &= \rho x_1^*(S_1 - x_1^*) + d(x_2^* - x_1^*) \\
0 &= \rho x_2^*(S_2 - x_2^*) + d(x_1^* - x_2)
\end{aligned}$$





(Total Population Equilibrium)

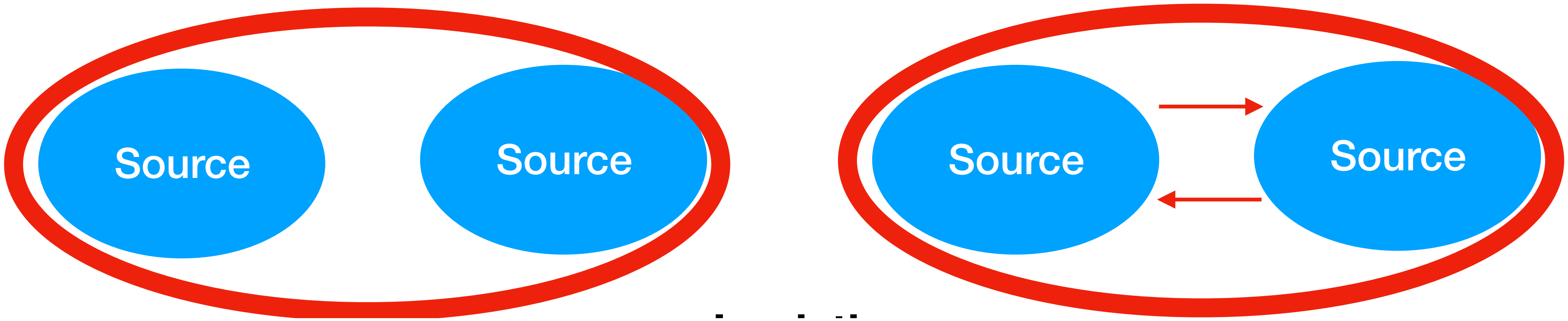
Logistique
Comparer les"TPE"

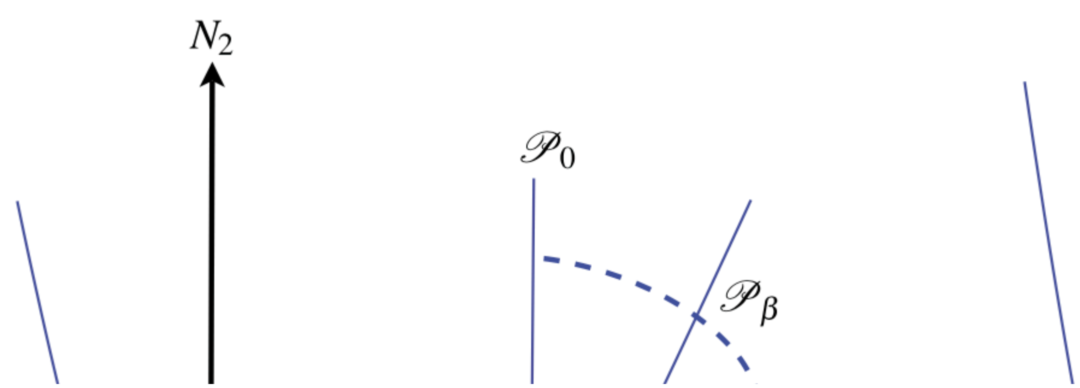
$$
\begin{aligned}
0 &= (S_1 - x_1^*) + d\frac{(x_2^* - x_1^*)}{\rho x_1^*} \\
0 &= (S_2 - x_2^*) + d\frac{(x_1^* - x_2^*)}{\rho x_2^*}
\end{aligned}
$$

$$
(S_1 + S_2) - (x_1^* + x_2^*) = \frac{d}{\rho}\left(\frac{(x_2^* - x_1^*)}{x_1^*} + \frac{(x_1^* - x_2^*)}{x_2^*}\right)
$$

$$
(S_1 + S_2) - (x_1^* + x_2^*) = \frac{d}{\rho}\left(\frac{(x_2^* - x_1^*)x_2^* + (x_1^* - x_2^*)x_1^*}{x_1^* x_2^*}\right)
$$

$$
(S_1 + S_2) - (x_1^* + x_2^*) = \frac{d}{\rho}\left(\frac{(x_2^* - x_1^*)^2}{x_1^* x_2^*}\right)
$$





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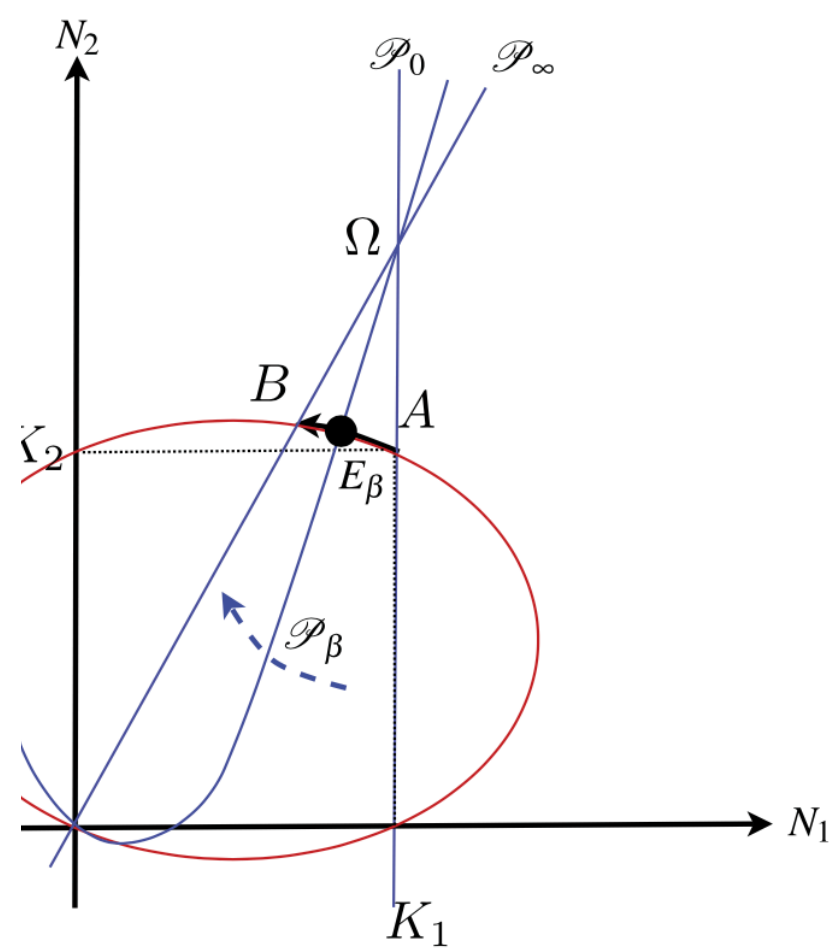
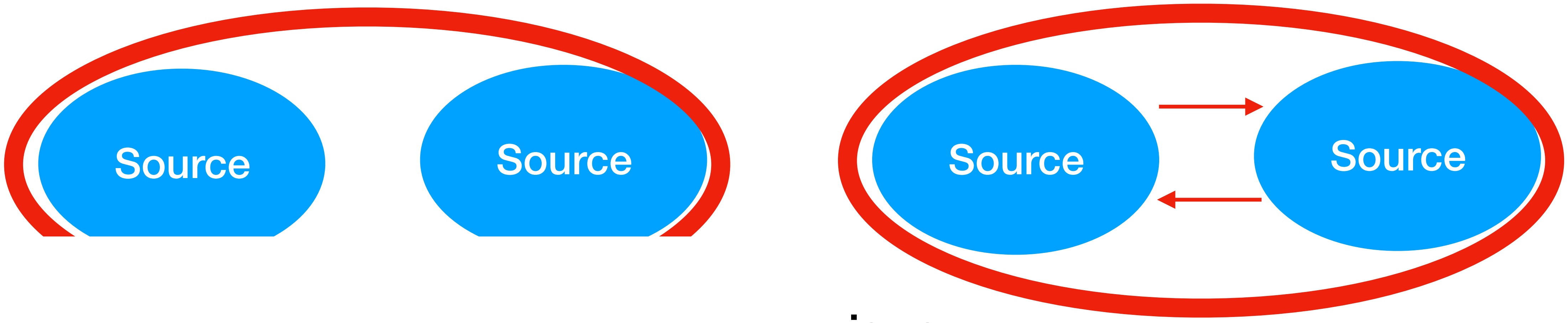
Volume 120, March 2018, Pages 11-15

Asymmetric dispersal in the multi-patch logistic equation

Roger Arditi [a, b], Claude Lobry [c], Tewfik Sari [d, e]



$$\begin{cases} \dfrac{dN_1}{dt} \\ \dfrac{dN_2}{dt} \end{cases}$$



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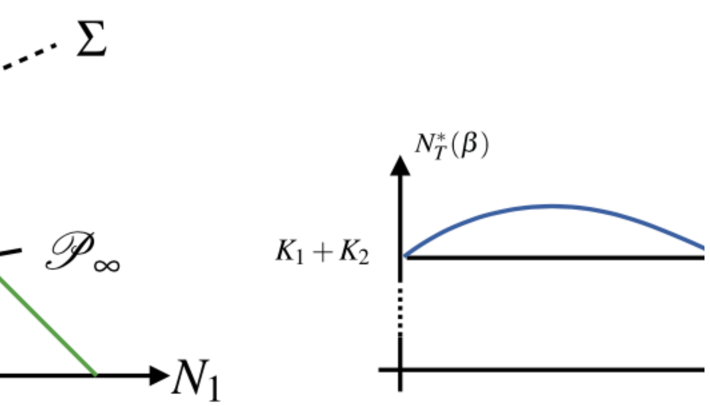

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TPB Theoretical Population Biology

Asymmetric dispersal in the multi-patch logistic equation

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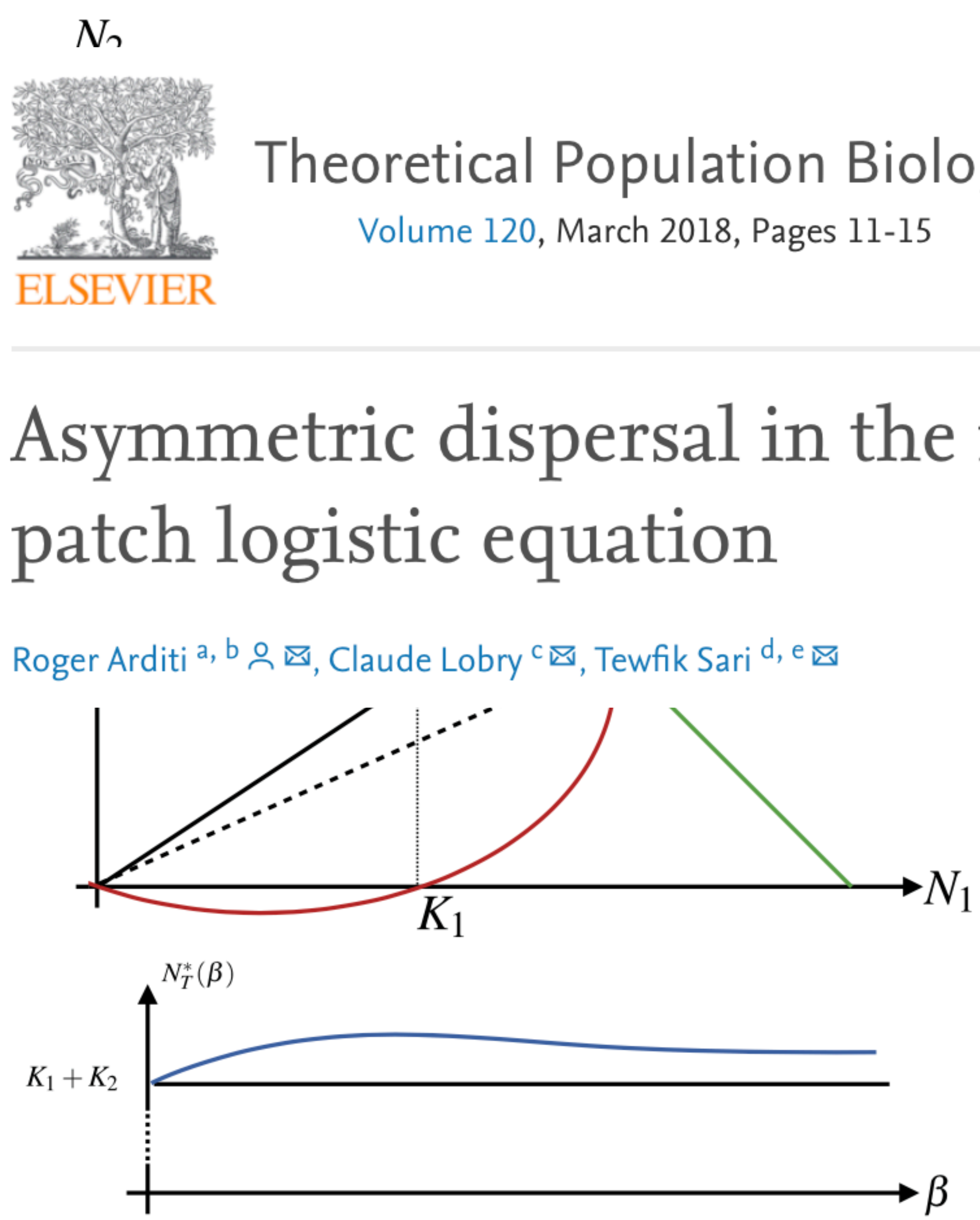
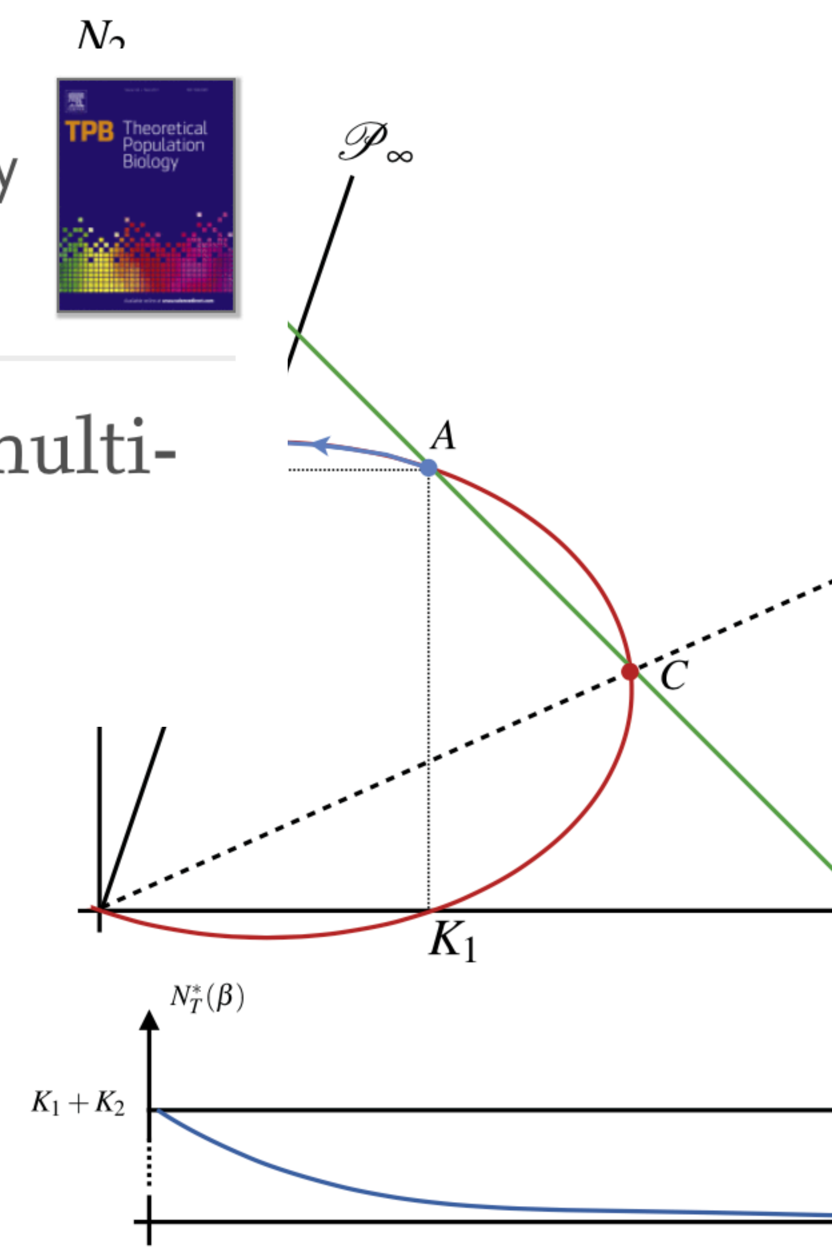
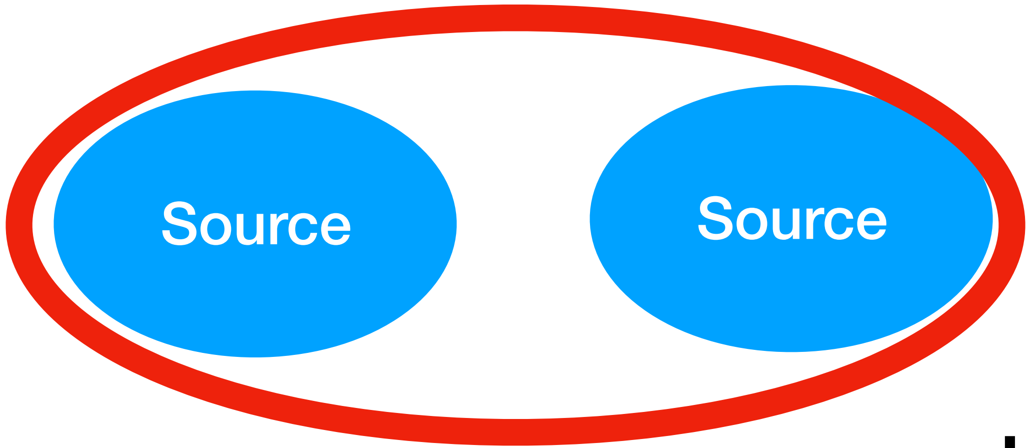
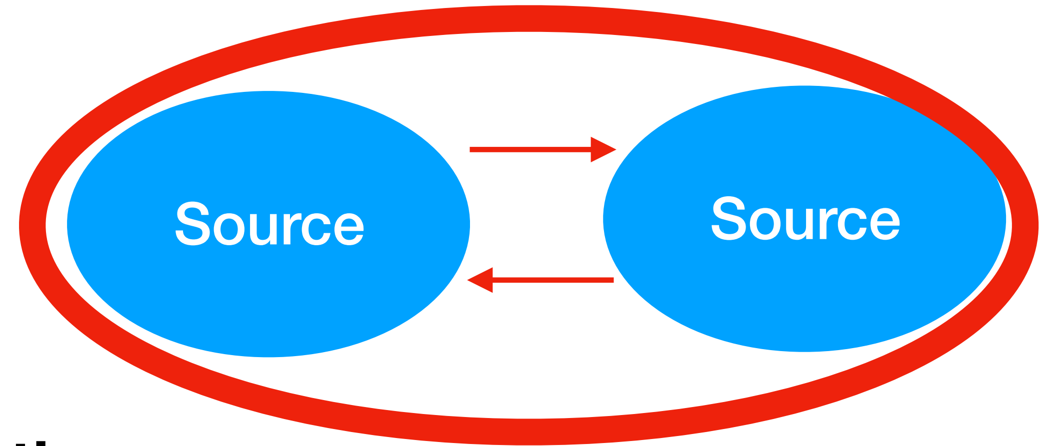






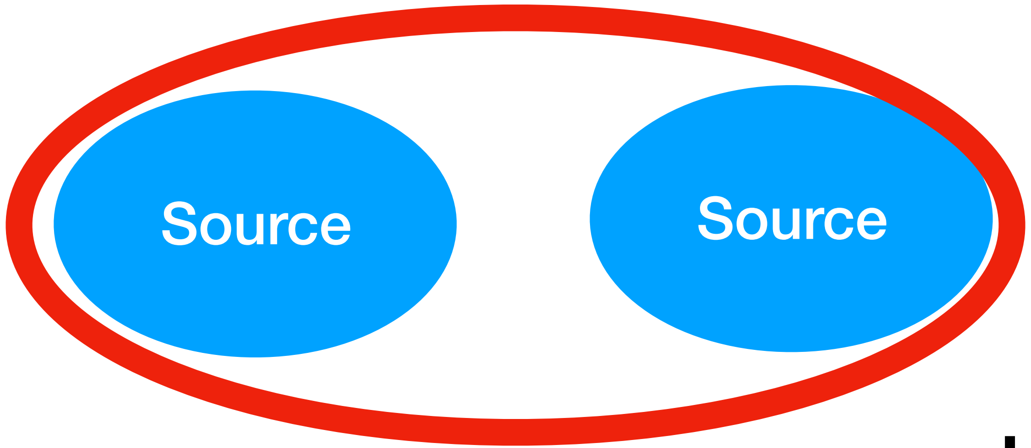


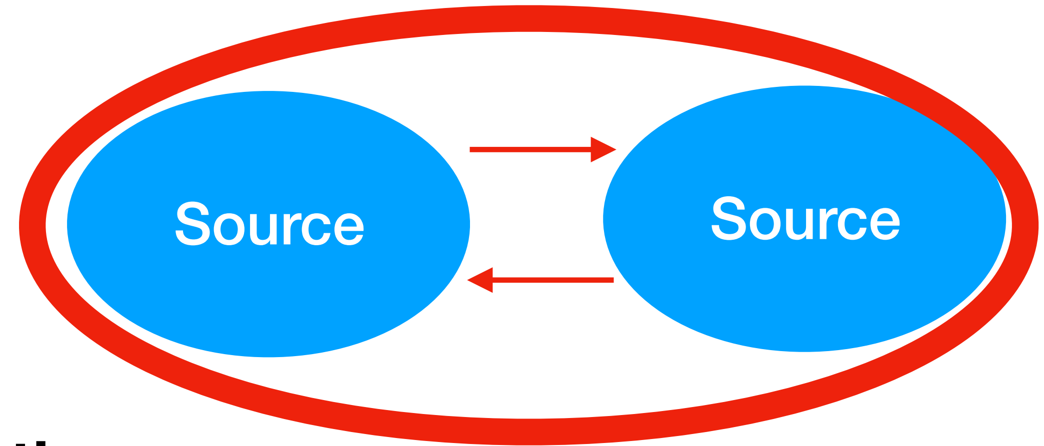
Logistique

(Total Population Equilibrium)

Comparer les"TPE"

$$\frac{dx}{dt} = r\,x\left(1 - \frac{x}{K}\right) = \frac{r}{K}x(K - x) = \rho x(S - x)$$





(Total Population Equilibrium)

Logistique
Comparer les"TPE"

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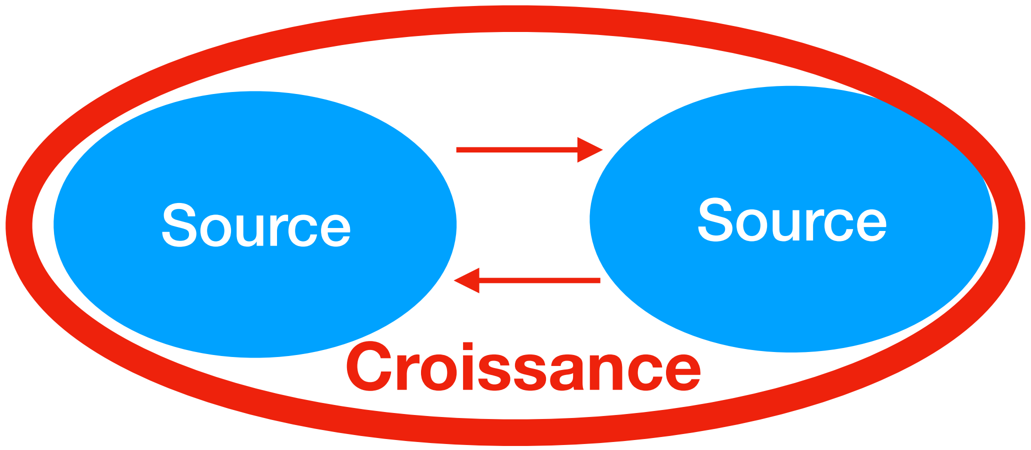
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$$\begin{cases} \dfrac{dN_1}{dt} = r_1 N_1 \left(1 - \dfrac{N_1}{K_1}\right) + \beta \left(N_2 - N_1\right) \\ \dfrac{dN_2}{dt} = r_2 N_2 \left(1 - \dfrac{N_2}{K_2}\right) + \beta \left(N_1 - N_2\right) \end{cases}$$

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2 puits (ou plus) en environnement constant ne donnent que des puits