

La relation
Ressource - Consommateur

III - Rosenzweig-MacArthur

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Rosenzweig-MacArthur

Rosenzweig 1941

Mac Arthur 1930-72

$$\dot{x} = f(x) - \frac{1}{Y} \mu(x, y) \cdot y$$

$$\dot{y} = \mu(x, y) \cdot y - \nu(y) \cdot y$$

Graphical Representation and Stability Conditions of Predator-Prey Interactions

M. L. Rosenzweig; R. H. MacArthur

American Naturalist, Vol. 97, No. 895 (Jul. - Aug., 1963), 209-223.

Stable URL:

<http://links.jstor.org/sici?sici=0003-0147%28196307%2F08%2997%3A895%3C209%3AGRASCO%3E2.0.CO%3B2-L>



Rosenzweig-MacArthur

Rosenzweig 1941

Mac Arthur 1930-72

$$\dot{x} = f(x) - \frac{1}{Y} \mu(x, y) \cdot y$$

$$\dot{y} = \mu(x, y) \cdot y - \nu(y) \cdot y$$

On va étudier le modèle attribué à Rosenzweig et Mac Arthur.

On va successivement :

- Faire une analyse par isoclines.
- Discuter sur la nature de la jacobienne en les équilibres.
- Montrer l'existence de cycles.....
- et d'un sévère problème "atto-fox" associé.

Rosenzweig-MacArthur

$$\dot{x} = f(x) - \frac{1}{Y} \mu(x, y) \cdot y$$

$$\dot{y} = \mu(x, y) \cdot y - \nu(y) \cdot y$$

- $f(x) = r \cdot x \left(1 - \frac{x}{K}\right)$

En l'absence du consommateur la dynamique de la ressource est une logistique. Pour le reste il est identique au modèle de Gause (sans le refuge pour la ressource)

Rosenzweig-MacArthur

$$\dot{x} = f(x) - \frac{1}{Y} \mu(x, y) \cdot y$$

$$\dot{y} = \mu(x, y) \cdot y - \nu(y) \cdot y$$

- $f(x) = r \cdot x \left(1 - \frac{x}{K}\right)$
- $\mu(x, y) = \frac{\mu \cdot x}{e + x}$
- $\nu(y) = m$

Rosenzweig-MacArthur

$$\begin{aligned}\dot{x} &= f(x) - \frac{1}{Y}\mu(x, y) \cdot y \\ \dot{y} &= \mu(x, y) \cdot y - \nu(y) \cdot y\end{aligned}$$

- $f(x) = r \cdot x \left(1 - \frac{x}{K}\right)$
- $\mu(x, y) = \frac{\mu \cdot x}{e + x}$
- $\nu(y) = m$

$$\begin{aligned}\dot{x} &= r \cdot x \cdot \left(1 - \frac{x}{K}\right) - \frac{1}{Y} \frac{\mu \cdot x}{e + x} \cdot y \\ \dot{y} &= \left(\frac{\mu \cdot x}{e + x} - d\right) \cdot y\end{aligned}$$

Etude mathématique du modèle de Rosenzweig-MacArthur

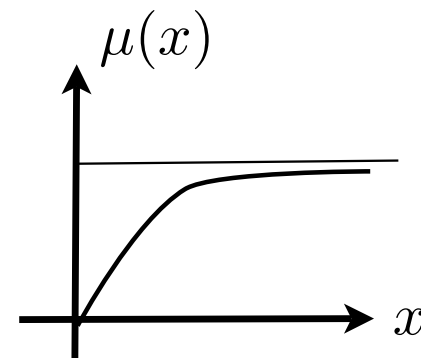
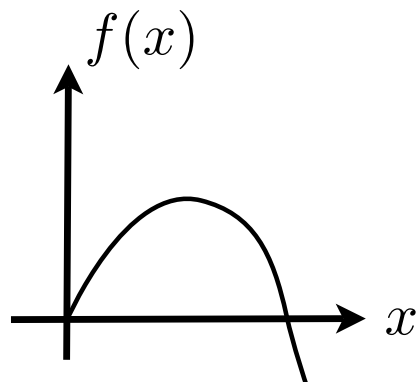
Au lieu de faire l'étude sur les fonctions explicites on prend des fonctions générales qui ont les mêmes propriétés qualitatives.

$$\dot{x} = r \cdot x \cdot \left(1 - \frac{x}{K}\right) - \frac{1}{Y} \frac{\mu \cdot x}{e + x} \cdot y$$

$$\dot{y} = \left(\frac{\mu \cdot x}{e + x} - m\right) \cdot y$$

$$\dot{x} = f(x) - \frac{1}{Y} \mu(x) \cdot y$$

$$\dot{y} = (\mu(x) - m) \cdot y$$



Etude mathématique du modèle de Rosenzweig-MacArthur

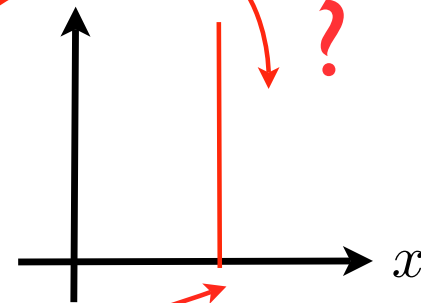
$$\begin{aligned} \dot{x} &= f(x) - \frac{1}{Y} \mu(x) \cdot y & \tilde{y} &= \frac{1}{Y} y \longrightarrow & \dot{x} &= f(x) - \mu(x) \cdot \tilde{y} \\ \dot{y} &= (\mu(x) - m) \cdot y & & & \dot{\tilde{y}} &= (\mu(x) - m) \cdot \tilde{y} \end{aligned}$$

$$\dot{x} = f(x) - \mu(x) \cdot y$$

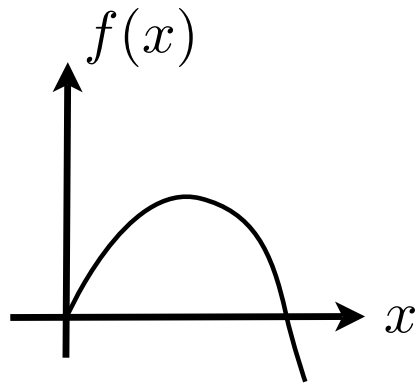
$$\dot{y} = (\mu(x) - m) \cdot y$$

$$\mu(x) = m$$

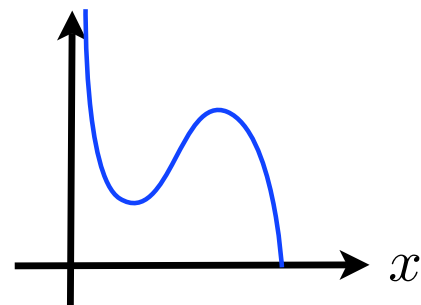
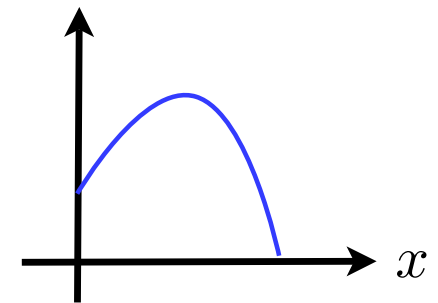
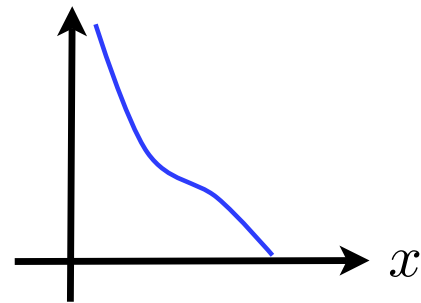
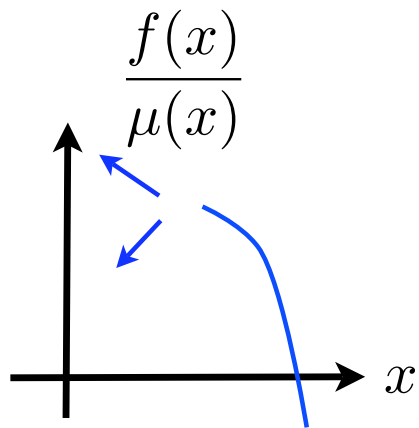
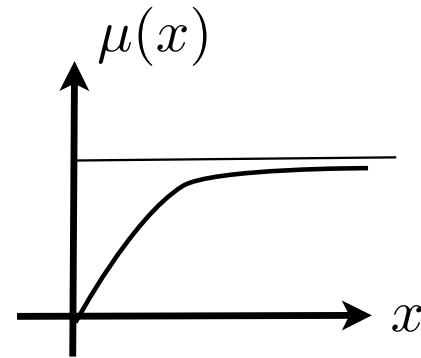
$$y = \frac{f(x)}{\mu(x)}$$



Etude mathématique du modèle de Rosenzweig-MacArthur



$$y = \frac{f(x)}{\mu(x)}$$

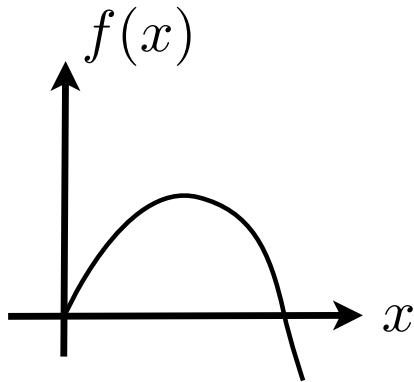


Etude mathématique du modèle de Rosenzweig-MacArthur

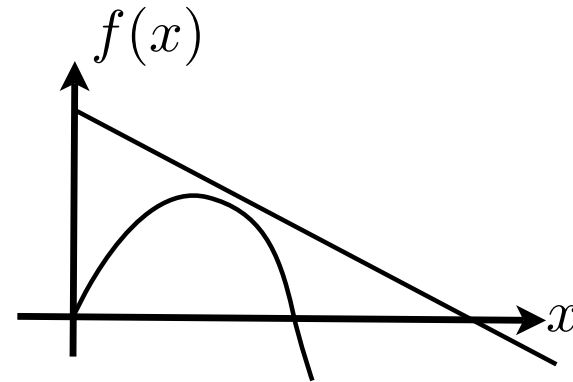
Les solutions sont bornées

$$\dot{x} = f(x) - \mu(x) \cdot y$$

$$\dot{y} = (\mu(x) - m) \cdot y$$



\Rightarrow



$$\Rightarrow f(x) < A - m \cdot x$$

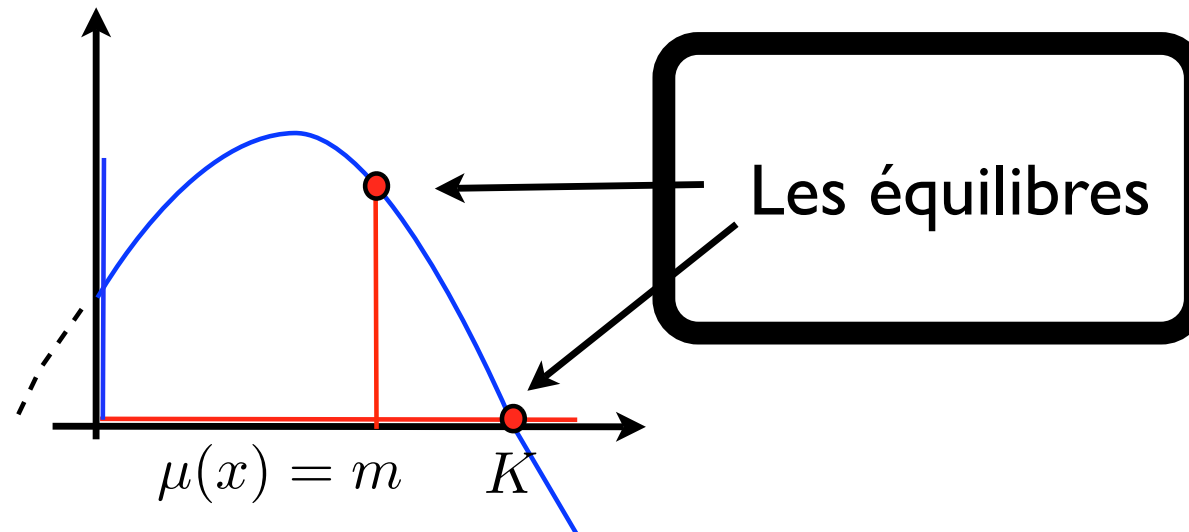
$$\underbrace{x + y}_{\dot{}} < A - m(x + y)$$

$$\Rightarrow (x(t) + y(t)) < A/m + e^{-mt}(x_o + y_o - A/m)$$

Etude mathématique du modèle de Rosenzweig-MacArthur

$$\begin{aligned}\dot{x} &= r \cdot x \cdot \left(1 - \frac{x}{K}\right) - \frac{1}{Y} \frac{\mu \cdot x}{e + x} \cdot y \\ \dot{\tilde{y}} &= \left(\frac{\mu \cdot x}{e + x} - d\right) \cdot y\end{aligned}\quad y = \frac{f(x)}{\mu(x)}$$

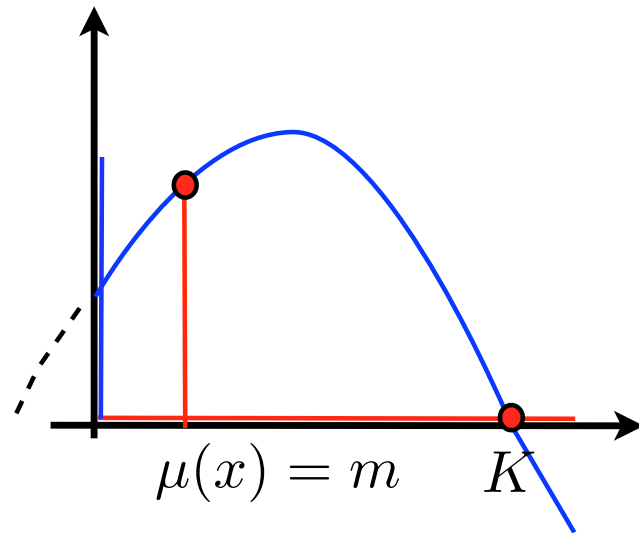
$$\frac{f(x)}{\mu(x)} = \frac{Y \cdot r}{\mu} \cdot (e + x) \cdot \left(1 - \frac{x}{K}\right)$$



Etude mathématique du modèle de Rosenzweig-MacArthur

$$\begin{aligned}\dot{x} &= r \cdot x \cdot \left(1 - \frac{x}{K}\right) - \frac{1}{Y} \frac{\mu \cdot x}{e + x} \cdot y \\ \dot{\tilde{y}} &= \left(\frac{\mu \cdot x}{e + x} - d\right) \cdot y\end{aligned}\quad y = \frac{f(x)}{\mu(x)}$$

$$\frac{f(x)}{\mu(x)} = \frac{Y \cdot r}{\mu} \cdot (e + x) \cdot \left(1 - \frac{x}{K}\right)$$



Selon m on
est à droite
ou à gauche
du maximum

Etude mathématique du modèle de Rosenzweig-MacArthur

$$\dot{x} = f(x) - \mu(x) \cdot y$$

$$\dot{y} = (\mu(x) - m) \cdot y$$

Stabilité des équilibres

$$\begin{cases} \dot{x} = f(x, y) \\ \dot{y} = g(x, y) \end{cases}$$

$$\left(\begin{array}{c|c} f'_x & f'_y \\ \hline g'_x & g'_y \end{array} \right)$$

On calcule la
jacobienne

Etude mathématique du modèle de Rosenzweig-MacArthur

$$\dot{x} = f(x) - \mu(x) \cdot y$$

$$\dot{y} = (\mu(x) - m) \cdot y$$

Stabilité des équilibres

$$\begin{cases} \dot{x} = f(x, y) \\ \dot{y} = g(x, y) \end{cases}$$

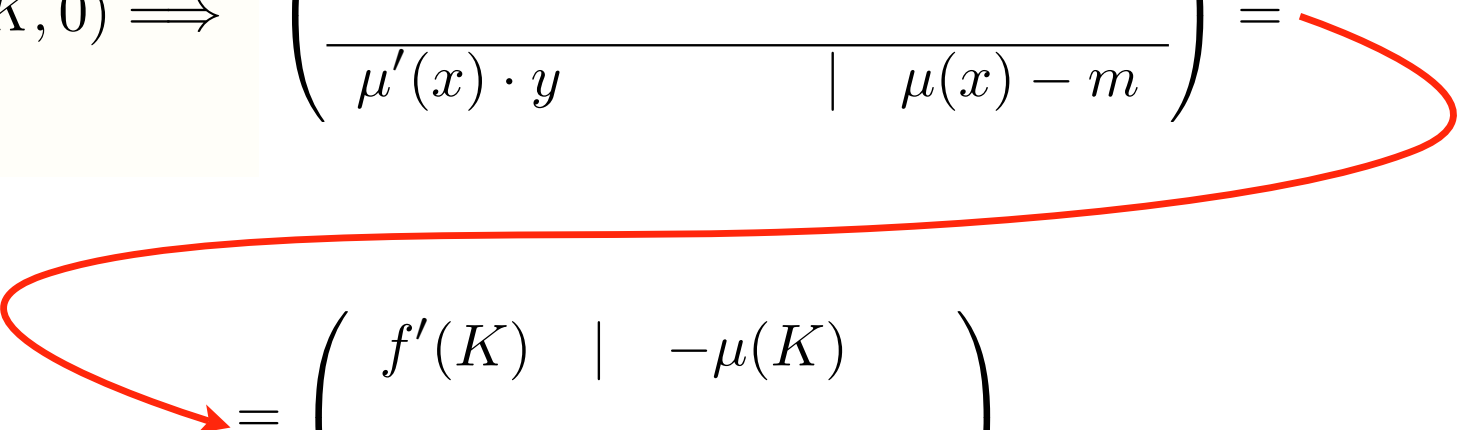
$$\left(\begin{array}{c|c} f'_x & f'_y \\ \hline g'_x & g'_y \end{array} \right) = \left(\begin{array}{c|c} f'(x) - \mu'(x) \cdot y & -\mu(x) \\ \hline \mu'(x) \cdot y & \mu(x) - m \end{array} \right)$$

Etude mathématique du modèle de Rosenzweig-MacArthur

$$\dot{x} = f(x) - \mu(x) \cdot y$$

$$\dot{y} = (\mu(x) - m) \cdot y$$

Stabilité des équilibres

$$(x, y) = (K, 0) \implies \left(\begin{array}{c|c} f'(x) - \mu'(x) \cdot y & -\mu(x) \\ \hline \mu'(x) \cdot y & \mu(x) - m \end{array} \right) =$$

$$= \left(\begin{array}{c|c} f'(K) & -\mu(K) \\ \hline 0 & \mu(K) - m \end{array} \right)$$

Etude mathématique du modèle de Rosenzweig-MacArthur

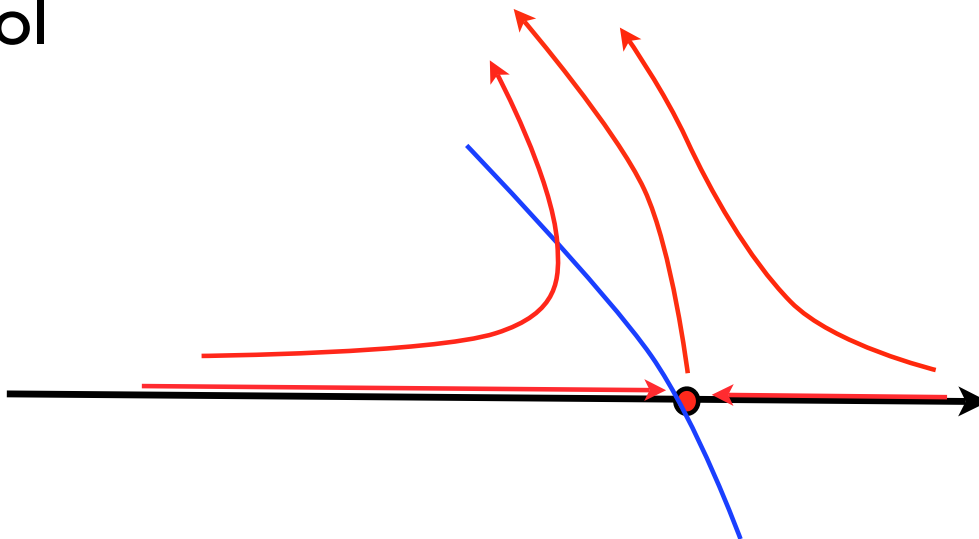
$$\dot{x} = f(x) - \mu(x) \cdot y$$

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Stabilité des équilibres

$$(x, y) = (K, 0) \implies \left(\begin{array}{c|c} f'(K) & -\mu(K) \\ \hline 0 & \mu(K) - m \end{array} \right) \implies \begin{cases} \lambda_1 = f'(K) < 0 \\ \lambda_2 = \mu(K) - m > 0 \end{cases}$$

Col



Etude mathématique du modèle de Rosenzweig-MacArthur

Stabilité des équilibres

$$\dot{x} = f(x) - \mu(x) \cdot y$$

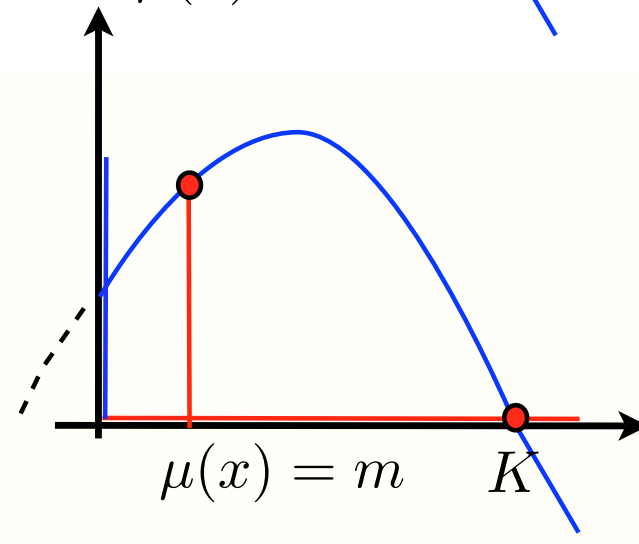
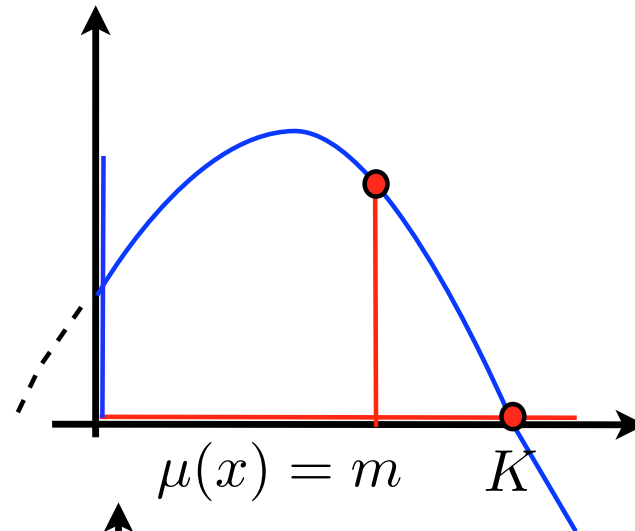
$$\dot{y} = (\mu(x) - m) \cdot y$$

$$x_e \iff \mu(x_e) = m$$

$$y_e = \frac{f(x_e)}{m}$$

$$\left(\begin{array}{c|c} f'(x) - \mu'(x) \cdot y & -\mu(x) \\ \hline \mu'(x) \cdot y & \mu(x) - m \end{array} \right) =$$

$$= \left(\begin{array}{c|c} f'(x_e) - \mu'(x_e) \cdot \frac{f(x_e)}{\mu(x_e)} & -m \\ \hline \mu'(x_e) \cdot \frac{f(x_e)}{\mu(x_e)} & 0 \end{array} \right)$$



Etude mathématique du modèle de Rosenzweig-MacArthur

$$\left(\begin{array}{c|c} f'(x_e) - \mu'(x_e) \cdot \frac{f(x_e)}{\mu(x_e)} & -m \\ \hline \mu'(x_e) \cdot \frac{f(x_e)}{\mu(x_e)} & 0 \end{array} \right)$$

$$\lambda^2 - \lambda \left(f'(x_e) - \mu'(x_e) \cdot \frac{f(x_e)}{\mu(x_e)} \right) + \mu'(x_e) \cdot f'(x_e) = 0$$

Ce transparent et les deux qui suivent détaillent un calcul qui permet de savoir si la matrice est stable ou non

Etude mathématique du modèle de Rosenzweig-MacArthur

$$\left(\begin{array}{c|c} f'(x_e) - \mu'(x_e) \cdot \frac{f(x_e)}{\mu(x_e)} & -m \\ \hline \mu'(x_e) \cdot \frac{f(x_e)}{\mu(x_e)} & 0 \end{array} \right)$$

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Etude mathématique du modèle de Rosenzweig-MacArthur

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$$\lambda^2 - \lambda \left(\mu(x_e) \frac{\mu(x_e) f'(x_e) - \mu'(x_e) \cdot f(x_e)}{\mu^2(x_e)} \right) + \mu'(x_e) \cdot f'(x_e) = 0$$

Etude mathématique du modèle de Rosenzweig-MacArthur

$$\left(\begin{array}{c|c} f'(x_e) - \mu'(x_e) \cdot \frac{f(x_e)}{\mu(x_e)} & -m \\ \hline \mu'(x_e) \cdot \frac{f(x_e)}{\mu(x_e)} & 0 \end{array} \right)$$

$$\lambda^2 - \lambda \left(f'(x_e) - \mu'(x_e) \cdot \frac{f(x_e)}{\mu(x_e)} \right) + \mu'(x_e) \cdot f'(x_e) = 0$$

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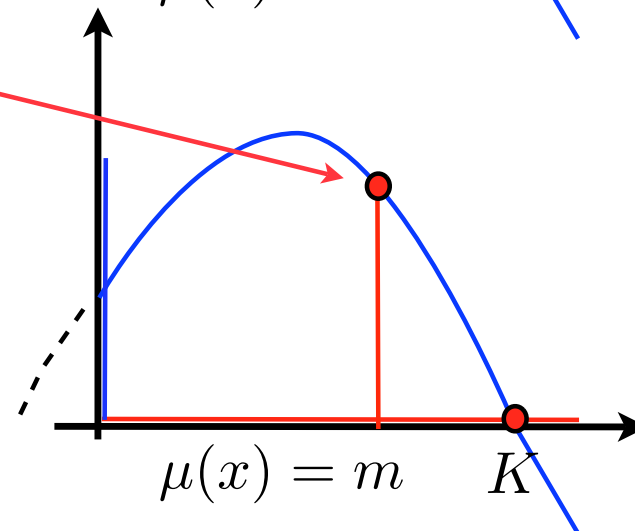
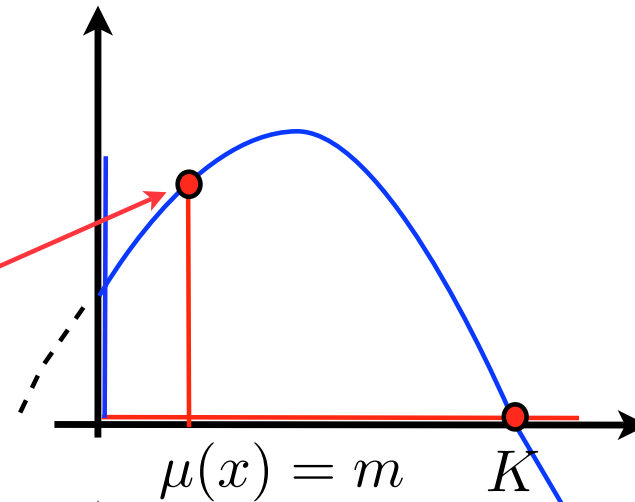
$$\lambda^2 - \lambda \left(\mu(x_e) \left(\frac{f}{\mu} \right)'(x_e) \right) + \mu'(x_e) \cdot f'(x_e) = 0$$

Etude mathématique du modèle de Rosenzweig-MacArthur

$$\lambda^2 - \lambda \left(\mu(x_e) \left(\frac{f}{\mu} \right)'(x_e) \right) + \mu'(x_e) \cdot f'(x_e) = 0$$

$\left(\frac{f}{\mu} \right)'(x_e) > 0 \implies \text{instable}$

$\left(\frac{f}{\mu} \right)'(x_e) < 0 \implies \text{stable}$



Etude mathématique du modèle de Rosenzweig-MacArthur

$$\lambda^2 - \lambda \left(\mu(x_e) \left(\frac{f}{\mu} \right)'(x_e) \right) + \mu'(x_e) \cdot f'(x_e) = 0$$

Remarque importante

Arguments reposant sur des propriétés qualitatives des fonctions etc...

- Calculs plus élégants

- Valable pour des “classes” de fonctions

Si l'on prend m comme paramètre lorsque l'on passe par le sommet de l'isocline les hypothèses du théorème de bifurcation de Hopf sont réalisées ce qui prouve l'existence de solutions périodiques. Dans la suite on prouve l'existence de solutions périodiques avec des arguments d'ordre de grandeur (perturbations singulières).

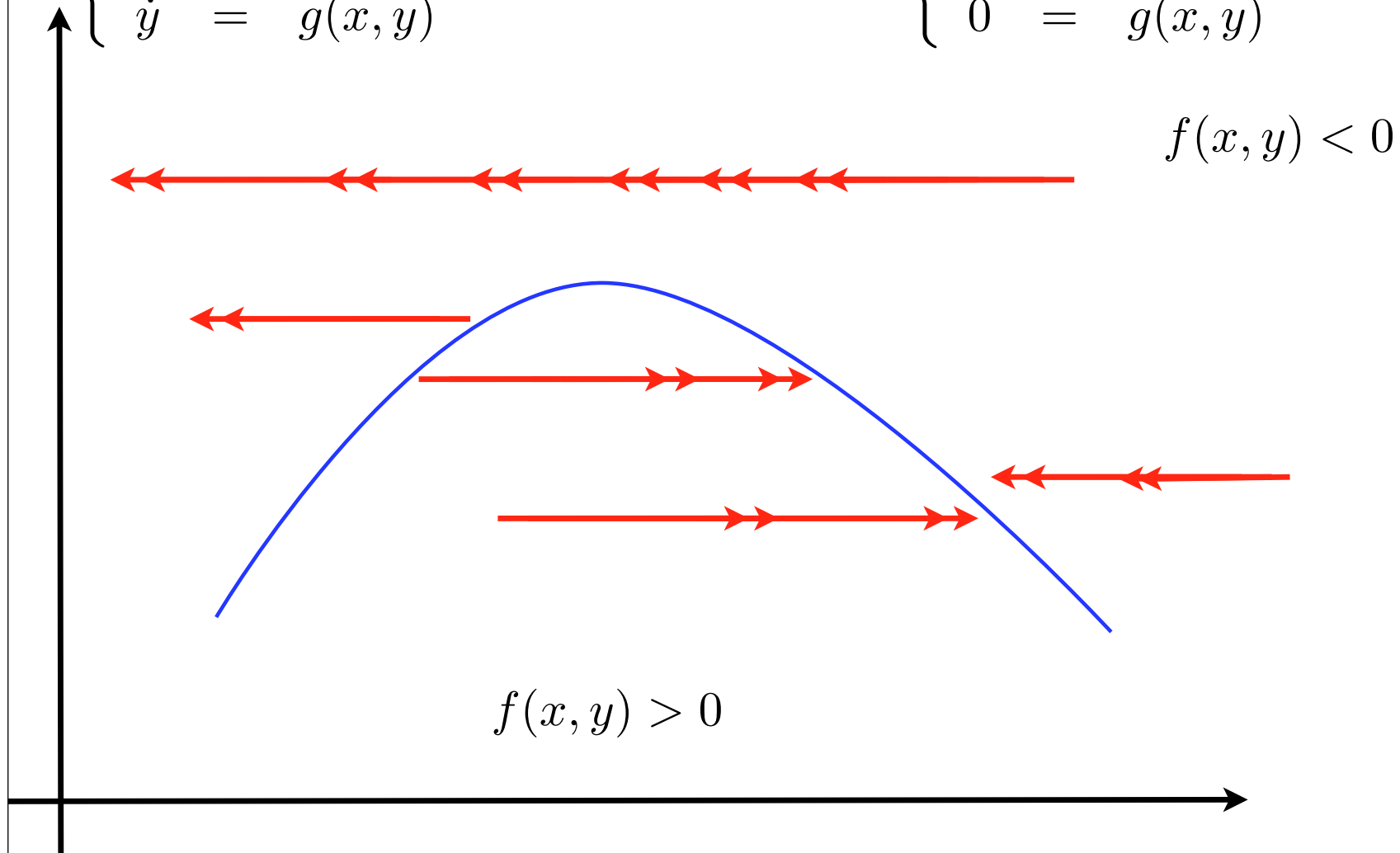
Intermède mathématique : Systèmes L-R

L-R = Lent-Rapide

$$\begin{cases} \dot{x} = \frac{1}{\varepsilon} f(x, y) \\ \dot{y} = g(x, y) \end{cases}$$

$$\varepsilon \approx 0$$

$$\begin{cases} 0 = f(x, y) \\ 0 = g(x, y) \end{cases}$$



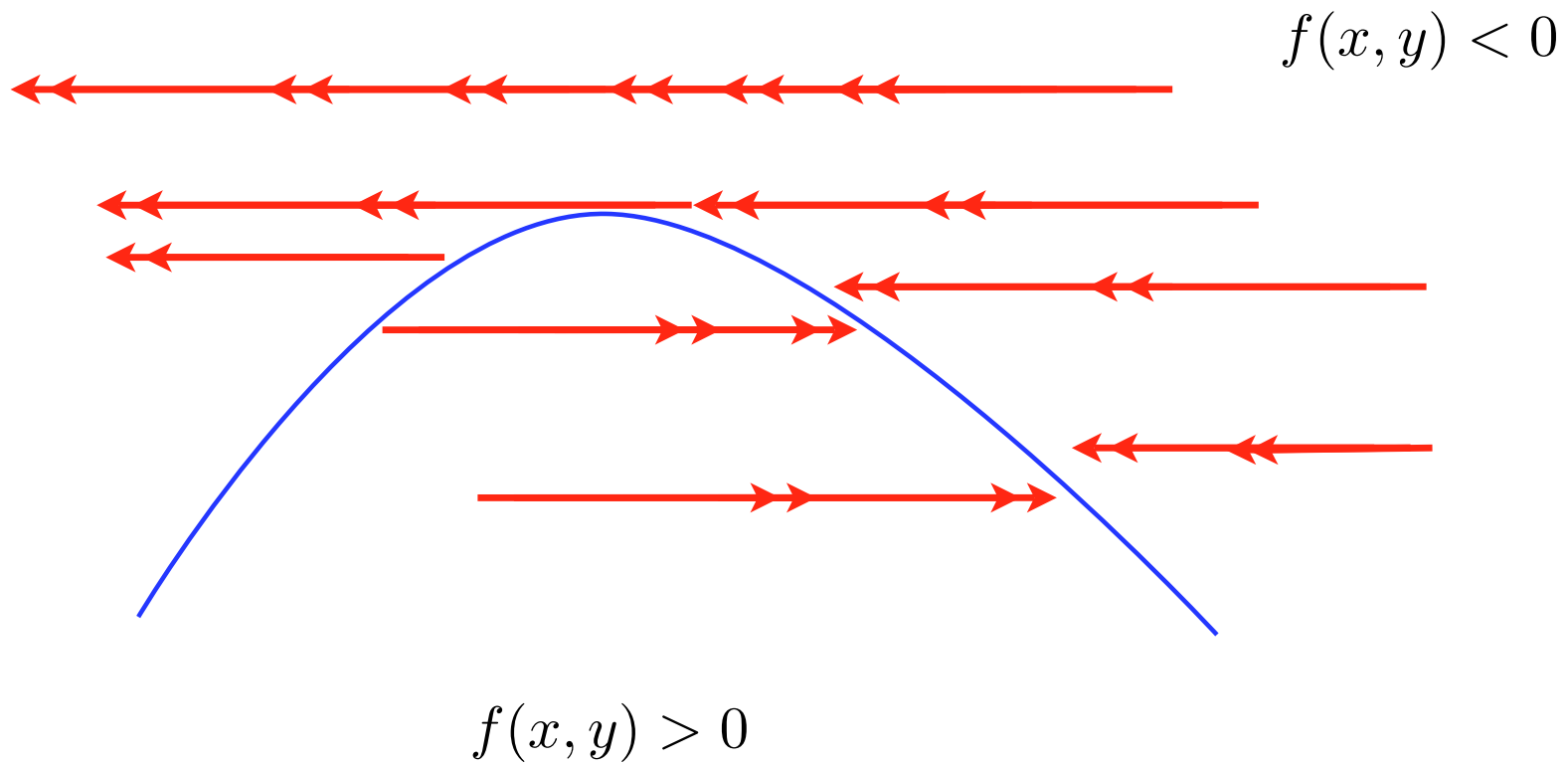
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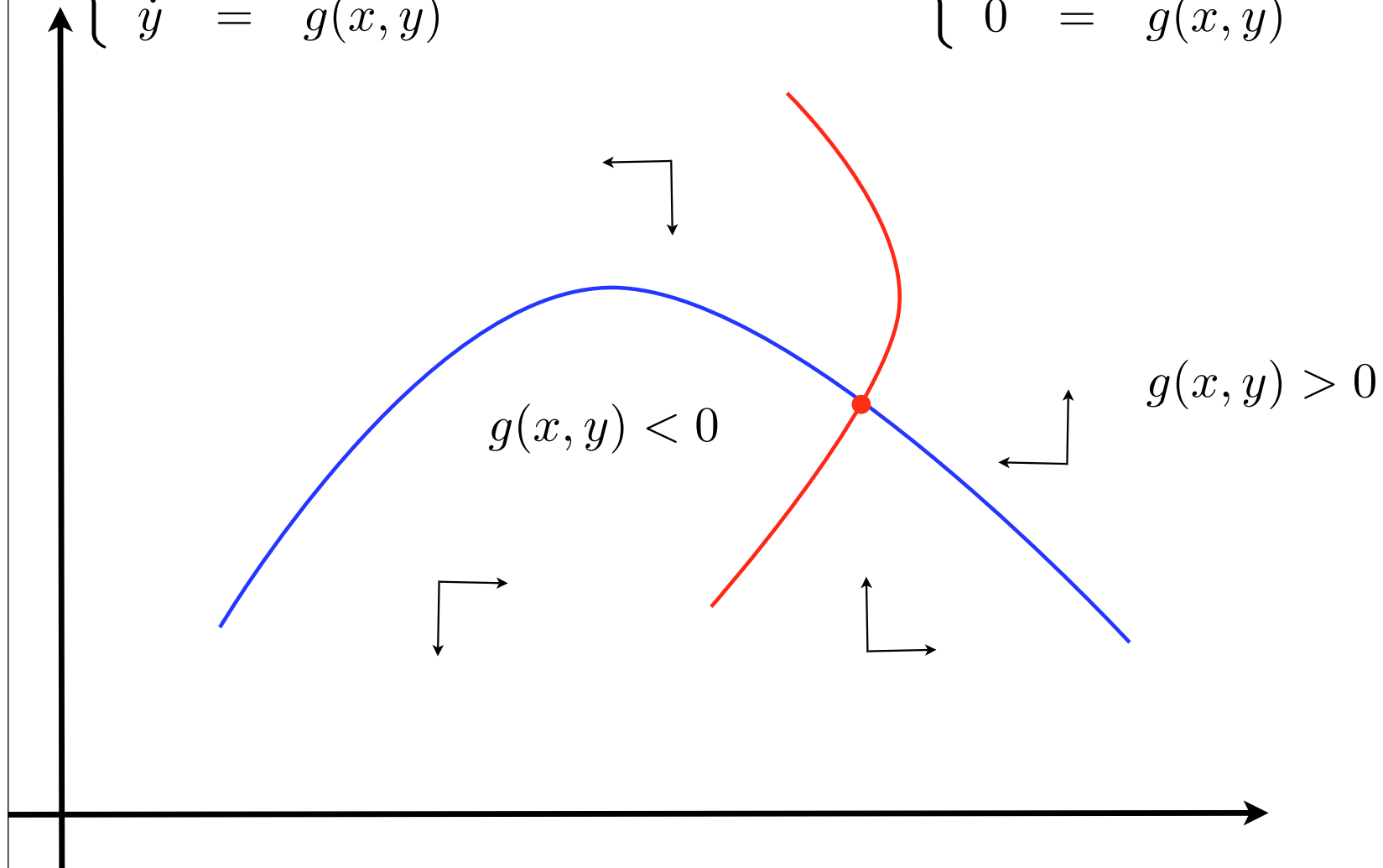
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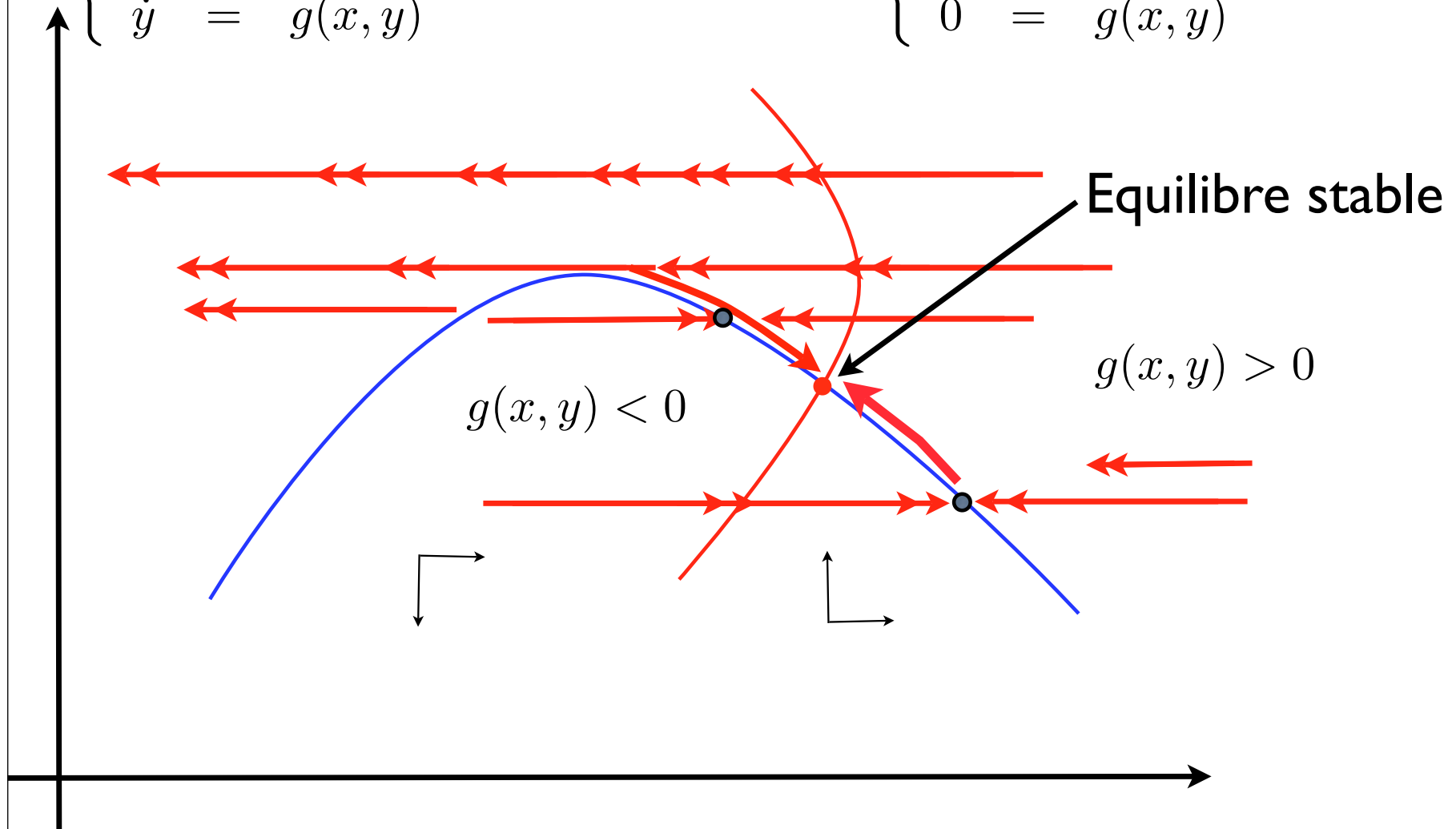
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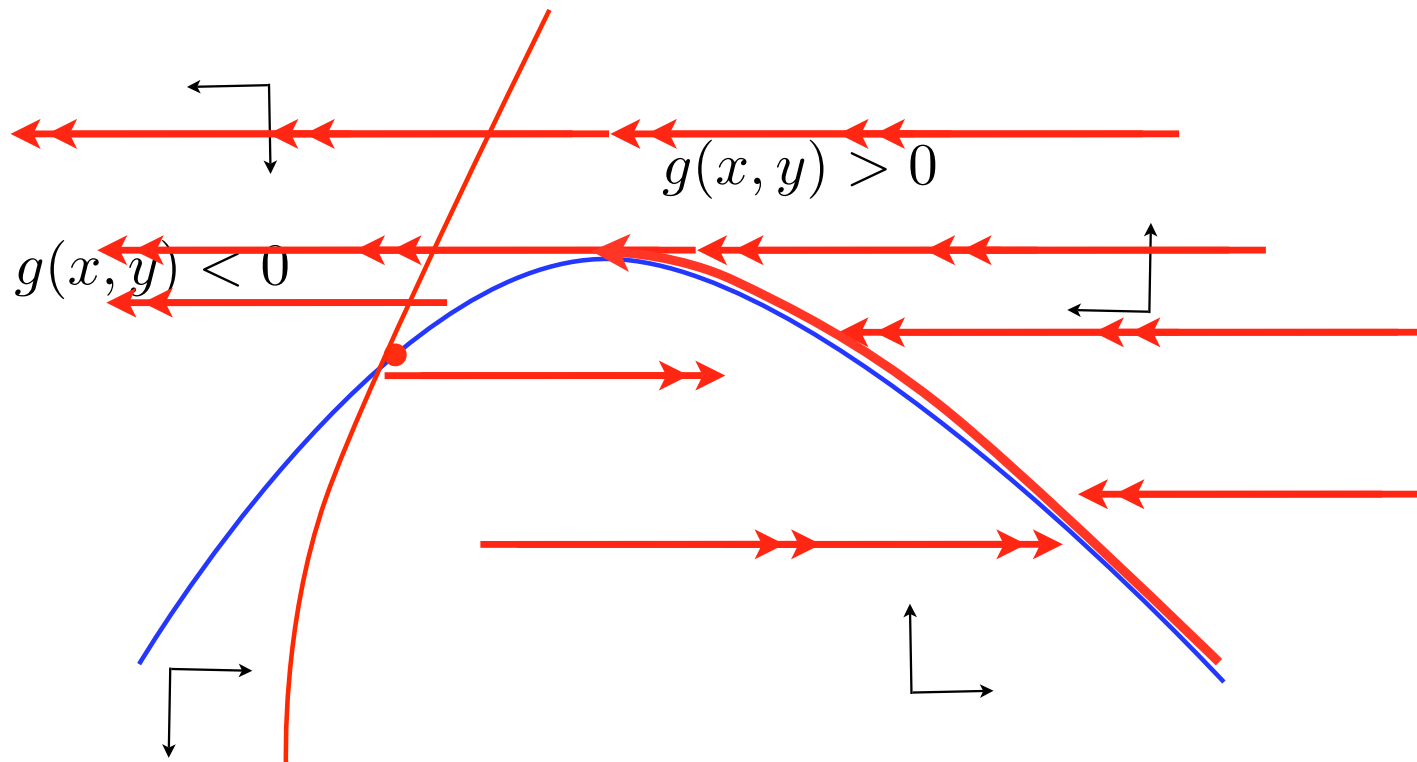
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Intermède mathématique : Systèmes L-R

L-R = Lent-Rapide

$$\begin{cases} \dot{x} = \frac{1}{\varepsilon} f(x, y) \\ \dot{y} = g(x, y) \end{cases} \quad \varepsilon \approx 0 \quad \begin{cases} 0 = f(x, y) \\ 0 = g(x, y) \end{cases}$$



Etude mathématique du modèle de Rosenzweig-MacArthur

Retour en arrière

Interaction

$$x(t + dt) = x(t) + dt \cdot f(x(t)) - dt \cdot c(x(t), y(t)) \cdot y(t)$$

$$y(t + dt) = y(t) + dt \cdot \mu(x(t), y(t)) \cdot y(t) - dt \cdot \nu(y(t)) \cdot y(t)$$

Une hypothèse forte :

$$\mu(x(t), y(t)) = Y \cdot c(x(t), y(t))$$

Proportionnalité entre ce qui est consommé et l'accroissement de la population de consommateurs

On remplace $c(x, y)$ par $\frac{1}{Y} \mu(x, y)$

Etude mathématique du modèle de Rosenzweig-MacArthur

On remplace $c(x, y)$ par $\frac{1}{Y} \mu(x, y)$

$$x(t + dt) = x(t) + dt \cdot \left(f(x(t)) - \frac{1}{Y} \mu(x(t), y(t)) \cdot y(t) \right)$$

$$y(t + dt) = y(t) + dt \cdot \left(\mu(x(t), y(t)) \cdot y(t) - \nu(y(t)) \cdot y(t) \right)$$

$$\frac{dx(t)}{dt} = f(x(t)) - \frac{1}{Y} \mu(x(t), y(t)) \cdot y(t)$$

$$\frac{dy(t)}{dt} = \mu(x(t), y(t)) \cdot y(t) - \nu(y(t)) \cdot y(t)$$

$$\dot{x} = f(x) - \frac{1}{Y} \mu(x, y) \cdot y$$

$$\dot{y} = \mu(x, y) \cdot y - \nu(y) \cdot y$$

Etude mathématique du modèle de Rosenzweig-MacArthur

On remplace $\mu(x, y)$ par $Y c(x, y)$

$$\dot{x} = f(x) - c(x, y) \cdot y$$

$$\dot{y} = (Y \cdot c(x, y) - \nu(y)) \cdot y$$

$$\dot{x} = f(x) - c(x) \cdot y$$

$$\dot{y} = (Y \cdot c(x) - m)y$$

$c(x)$ noté $\mu(x)$

$$\dot{x} = f(x) - \mu(x) \cdot y$$

$$\dot{y} = (\varepsilon \cdot \mu(x) - m)y$$

$$m < \varepsilon \cdot \mu(x)$$

$$m = \varepsilon \cdot \delta$$

$$\dot{x} = f(x) - \mu(x) \cdot y$$

$$\dot{y} = \varepsilon \cdot (\mu(x) - \delta) \cdot y$$

\iff

$$\dot{x} = \frac{1}{\varepsilon} (f(x) - \mu(x) \cdot y)$$

$$\dot{y} = (\mu(x) - \delta) \cdot y$$

Etude mathématique du modèle de Rosenzweig-MacArthur

Simulations de RMA

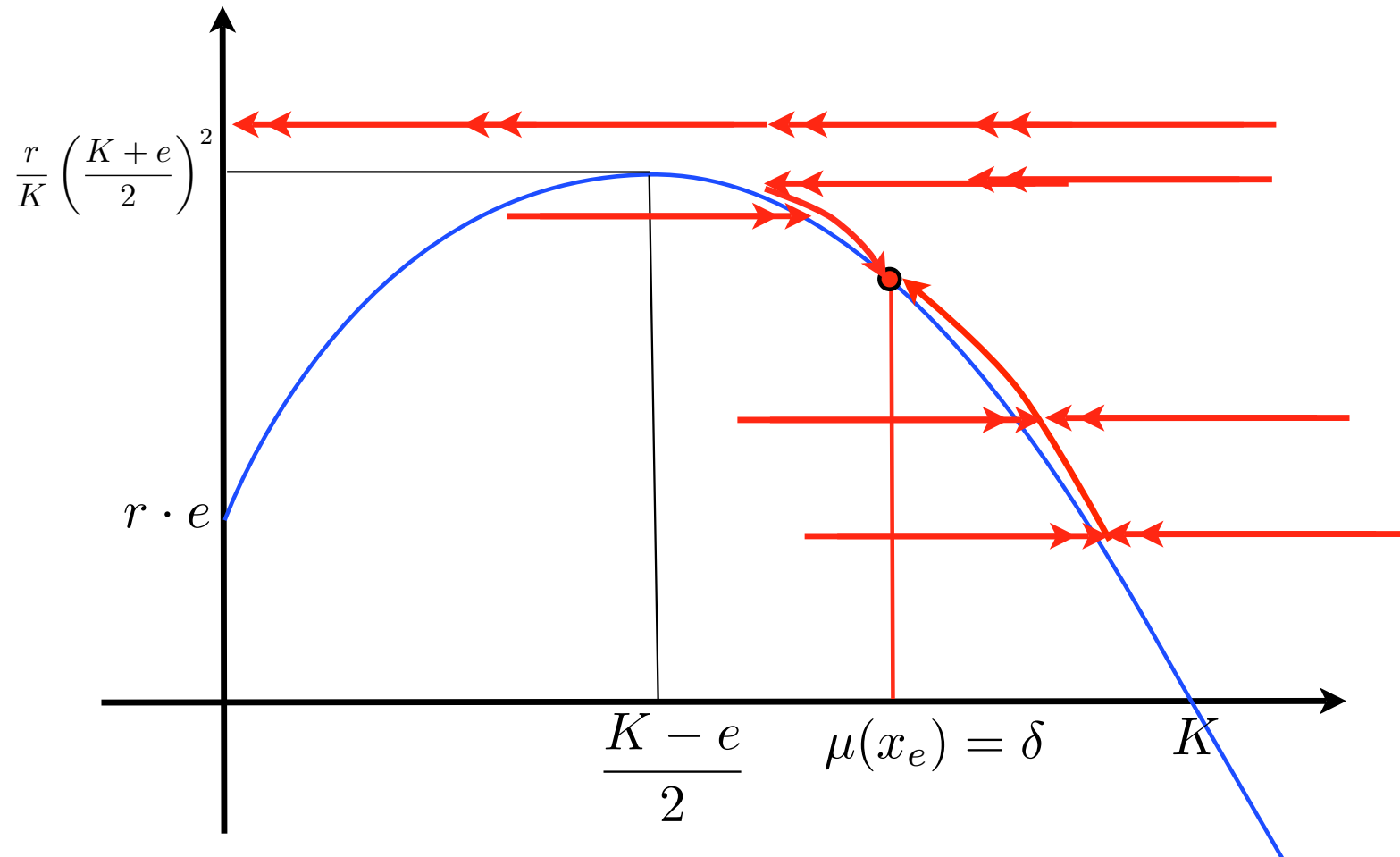
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$$\dot{x} = \frac{1}{\varepsilon} (f(x) - \mu(x) \cdot y)$$

- $f(x) = r \cdot x \cdot (1 - x/K)$

$$\dot{y} = (\mu(x) - \delta) \cdot y$$

- $\mu(x) = \frac{\mu \cdot x}{e + x}$



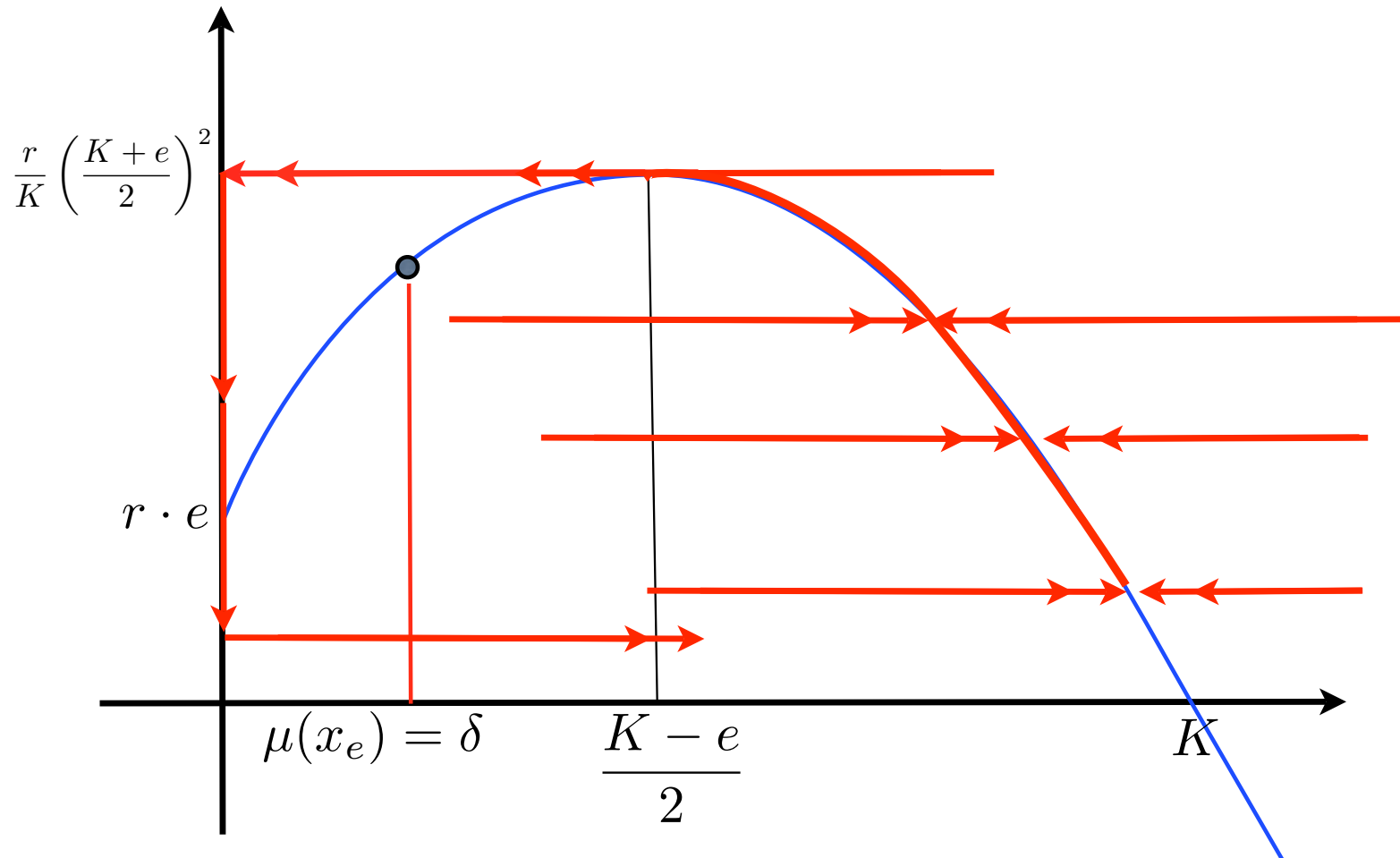
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$$\dot{x} = \frac{1}{\varepsilon} (f(x) - \mu(x) \cdot y)$$

- $f(x) = r \cdot x \cdot (1 - x/K)$

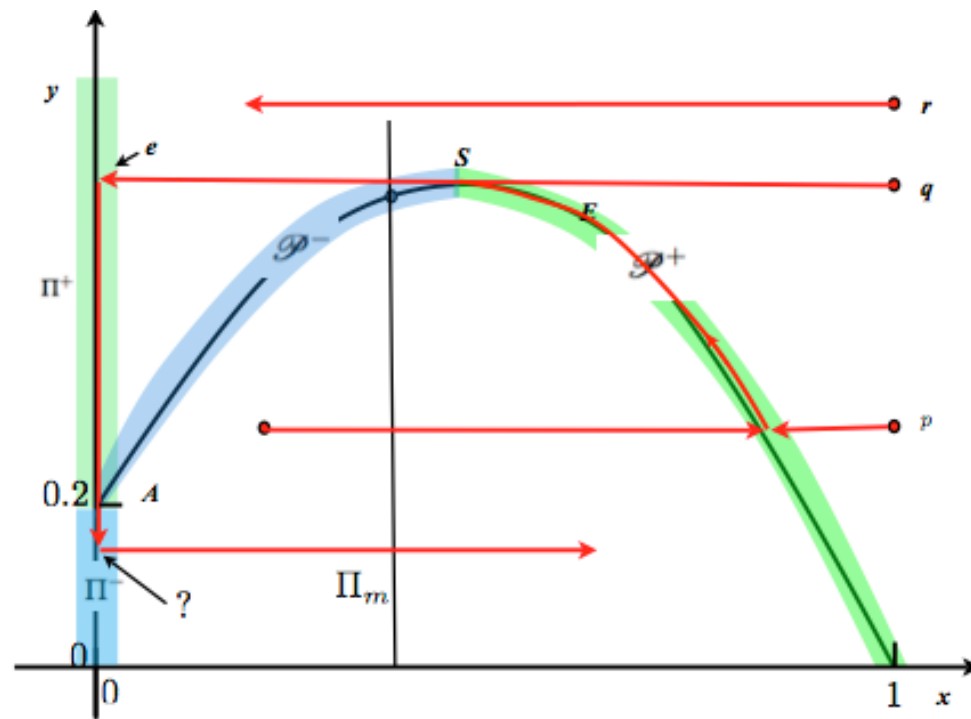
$$\dot{y} = (\mu(x) - \delta) \cdot y$$

- $\mu(x) = \frac{\mu \cdot x}{e + x}$

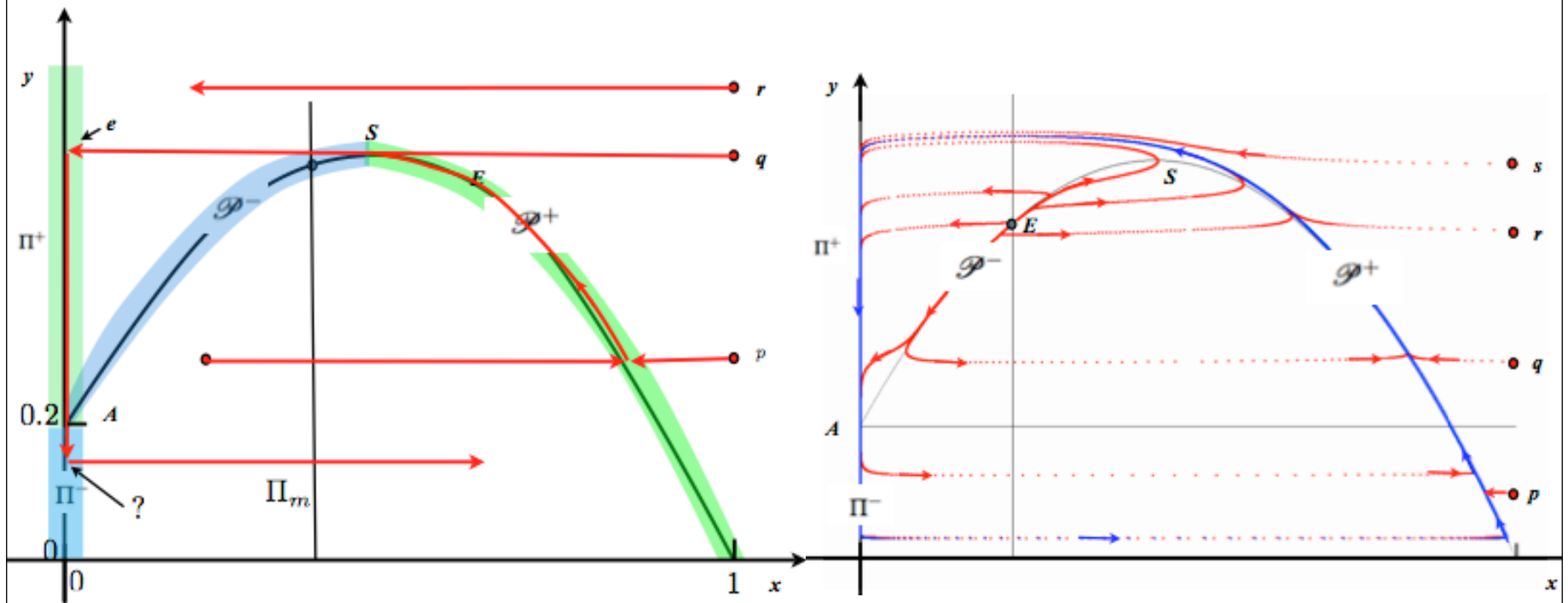


Simulations du modèle de Rosenzweig-MacArthur

$$\begin{cases} \frac{dx}{dt} = \frac{1}{\varepsilon} \left(2x(1-x) - \frac{x}{0.1+x}y \right) \\ \frac{dy}{dt} = \left(\frac{x}{0.1+x} - \delta \right) y \end{cases}$$



Simulations du modèle de Rosenzweig-MacArthur



Simulations du modèle de Rosenzweig-MacArthur

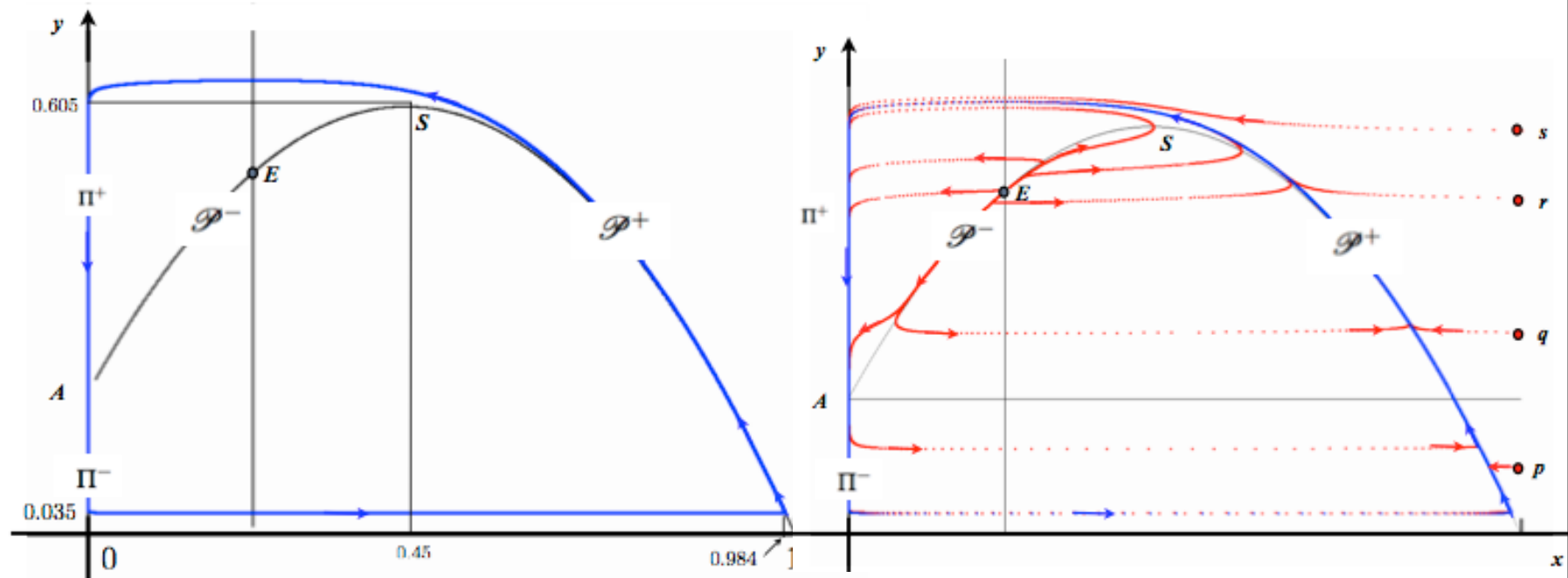


Figure 3: Limit cycle of (3) : $\varepsilon = 0.05$, $m : 0.7$

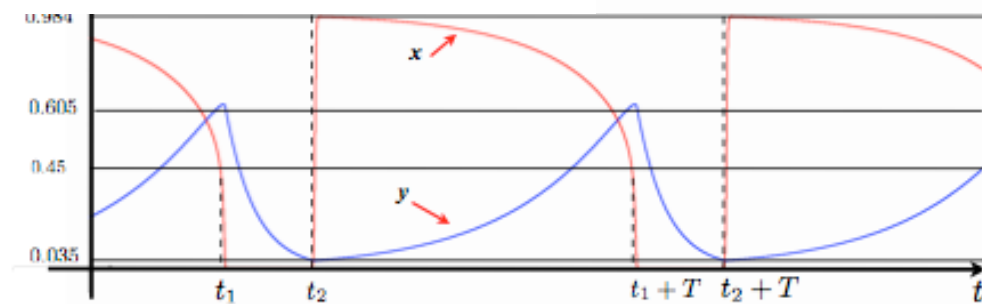
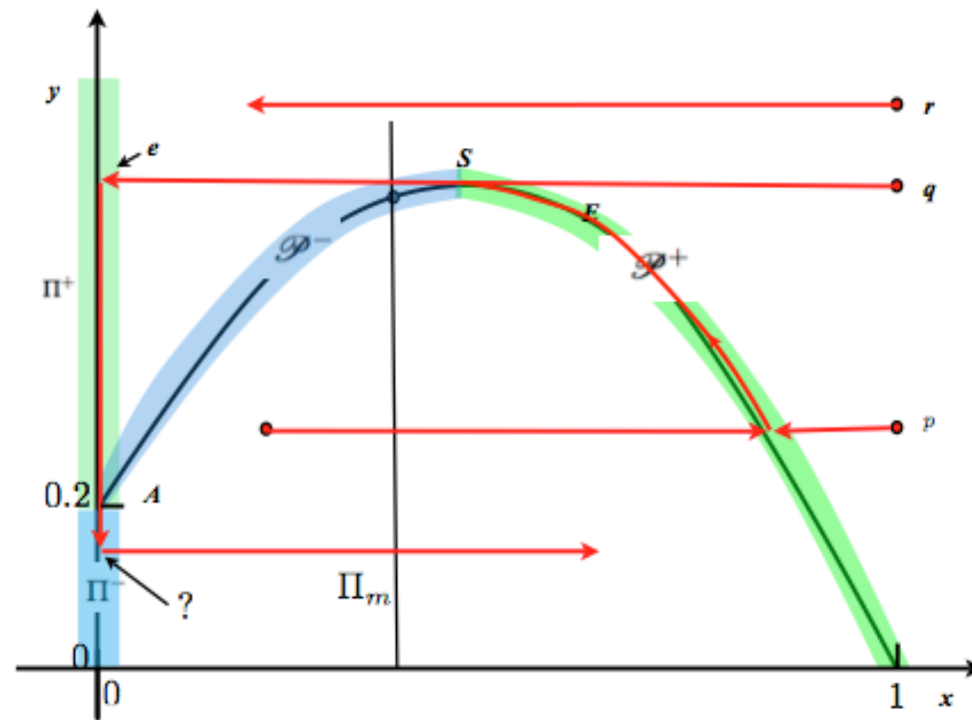


Figure 4: Limit cycle of (3); "jumps" of $x(t)$: $\varepsilon = 0.05$, $m : 0.7$

Simulations du modèle de Rosenzweig-MacArthur



y_0	0.4	0.5	0.6	0.7
$\exp(\frac{\xi_{min}}{\epsilon})$	$1.66 \cdot 10^{-9}$	$1.79 \cdot 10^{-16}$	$2.14 \cdot 10^{-24}$	$5.34 \cdot 10^{-33}$
x_{min}	$1.62 \cdot 10^{-9}$	$1.69 \cdot 10^{-16}$	$1.90 \cdot 10^{-24}$	$4.31 \cdot 10^{-33}$

Table 2: Values of the parameters : $\epsilon = 0.05$; $m = 0.7$; $x_0 = 0.05$; $\xi_0 = \epsilon \log(x_0)$

$$\begin{cases} \frac{dx}{dt} = xg(x) - h(x,y)y \\ \frac{dy}{dt} = \varepsilon h(x,y)y - \delta y \end{cases} \quad \begin{cases} \frac{dx}{dt} = \frac{1}{\varepsilon}(xg(x) - h(x,y)y) \\ \frac{dy}{dt} = (h(x,y) - m)y \end{cases}$$

$$\begin{cases} \frac{dx}{dt} = \frac{1}{\varepsilon} \left(2x(1-x) - \frac{xy}{0.1+x} \right) \\ \frac{dy}{dt} = \left(\frac{x}{0.1+x} - m \right) y \end{cases}$$

$$\begin{cases} \frac{dx}{dt} = \frac{1}{\varepsilon} \left(2x(1-x) - \frac{xy}{0.1+x} \right) \\ \frac{dy}{dt} = \left(\frac{x}{0.1+x} - m \right) y \end{cases}$$

Le modèle (3) ci-dessus possède pour des valeurs convenables des paramètres, comme par exemple :

$$m = 0.7 ; \varepsilon = 0.05$$

un cycle limite globalement asymptotiquement stable (G.A.S.). Donc, d'un point de vue mathématique, il est persistant. Toutefois, si nous faisons une simulation de ces équations à partir des conditions initiales :

$$x(0) = 0.1 ; y(0) = 0.5$$

nous observerons que la valeur de $x(t)$ va diminuer jusqu'à des quantités de l'ordre de 10^{-26} !! (Nous reviendrons longuement au § suivant sur les raisons de ce phénomène). Pour une telle valeur $x(t)$ ne représente plus rien. En effet supposons que une unité de x représente 10^8 individus (par exemple des renards) la valeur 10^{-26} représenterait un 10^{-18} ème de renard, un "atto-renard"¹.

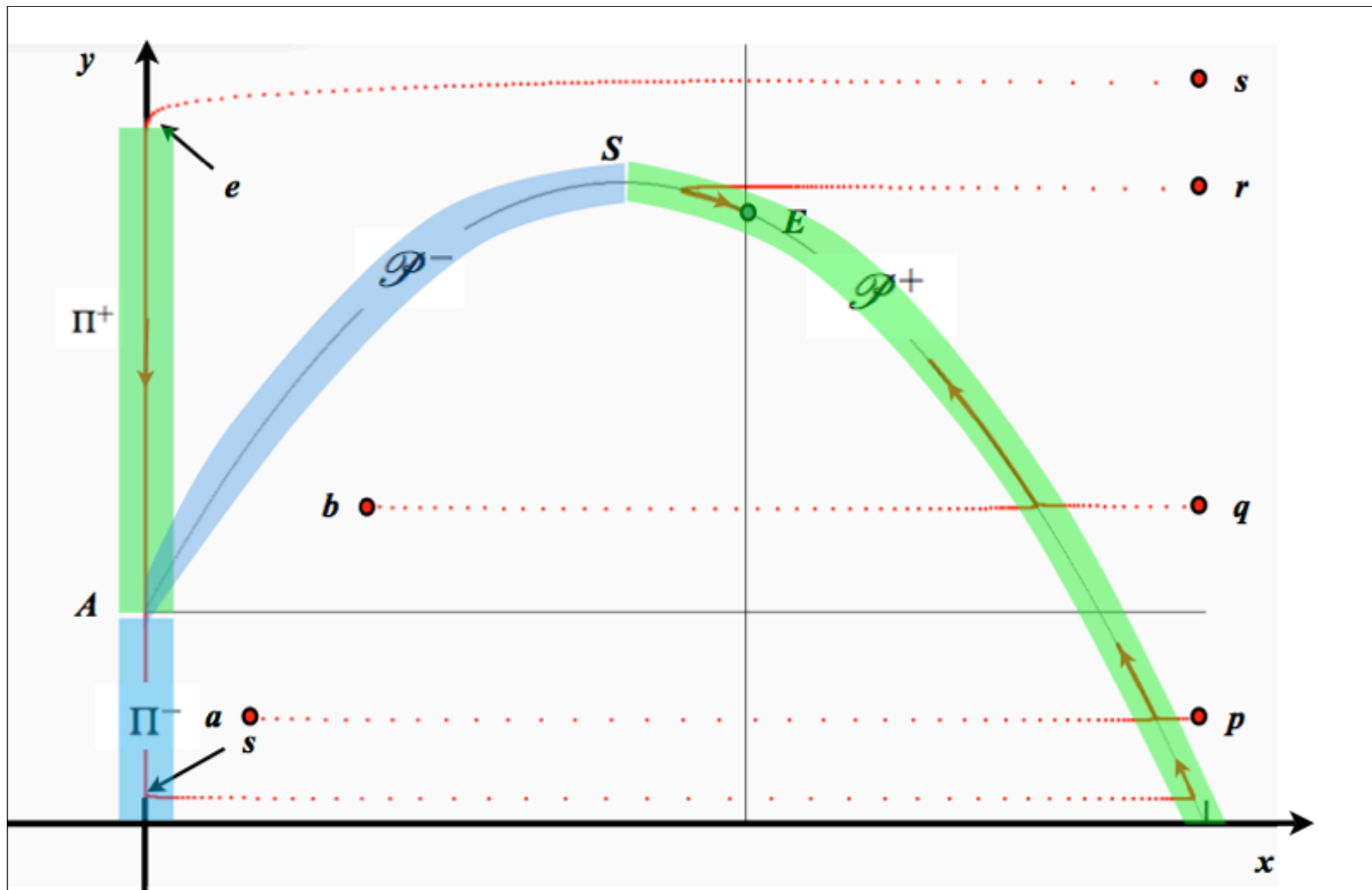


FIG. 2 – Quelques trajectoires avec $\varepsilon = 0.05$ et $m = 0.85$.

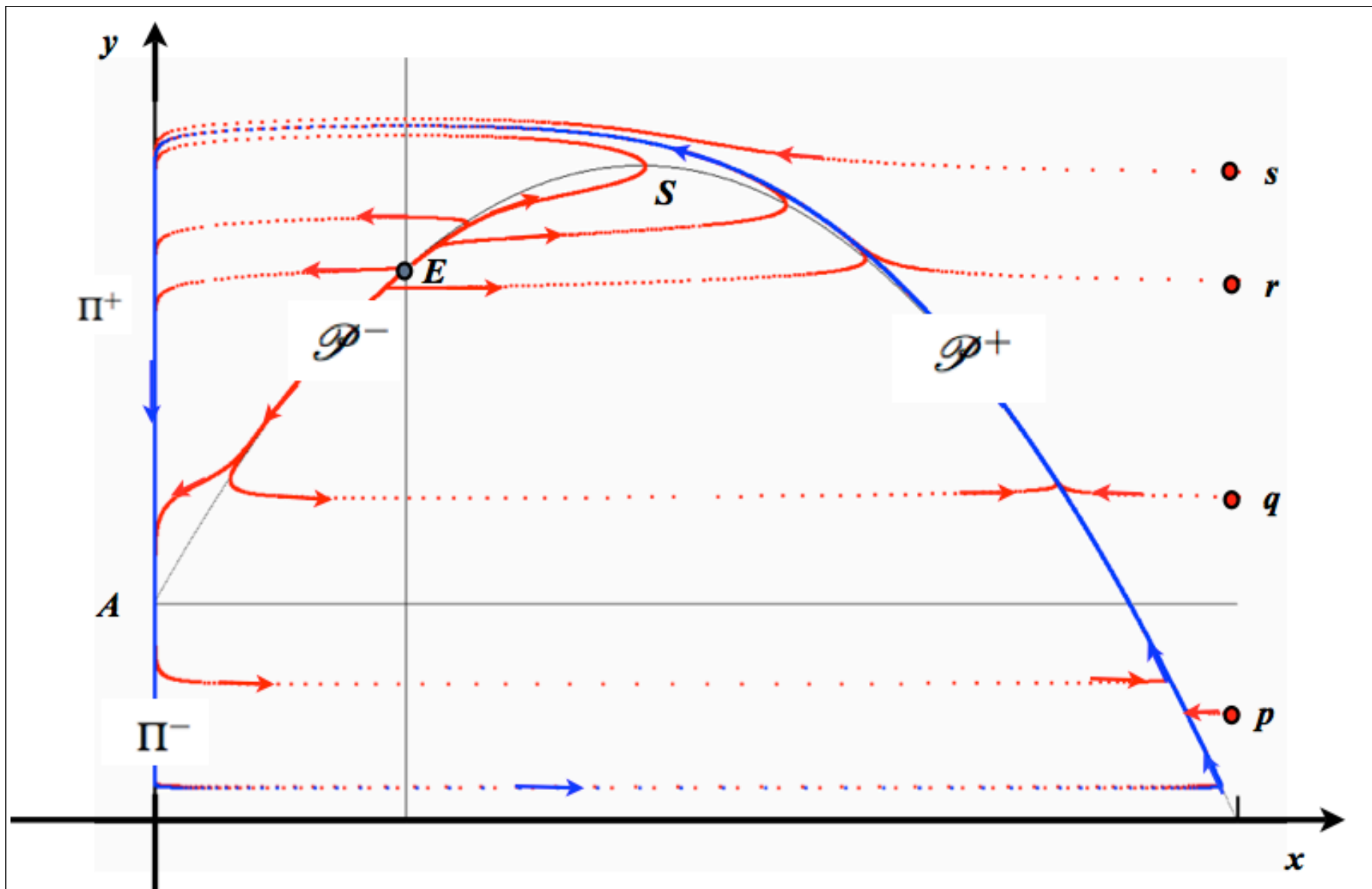


FIG. 3 – Quelques trajectoires avec $\varepsilon = 0.05$ et $m = 0.7$.

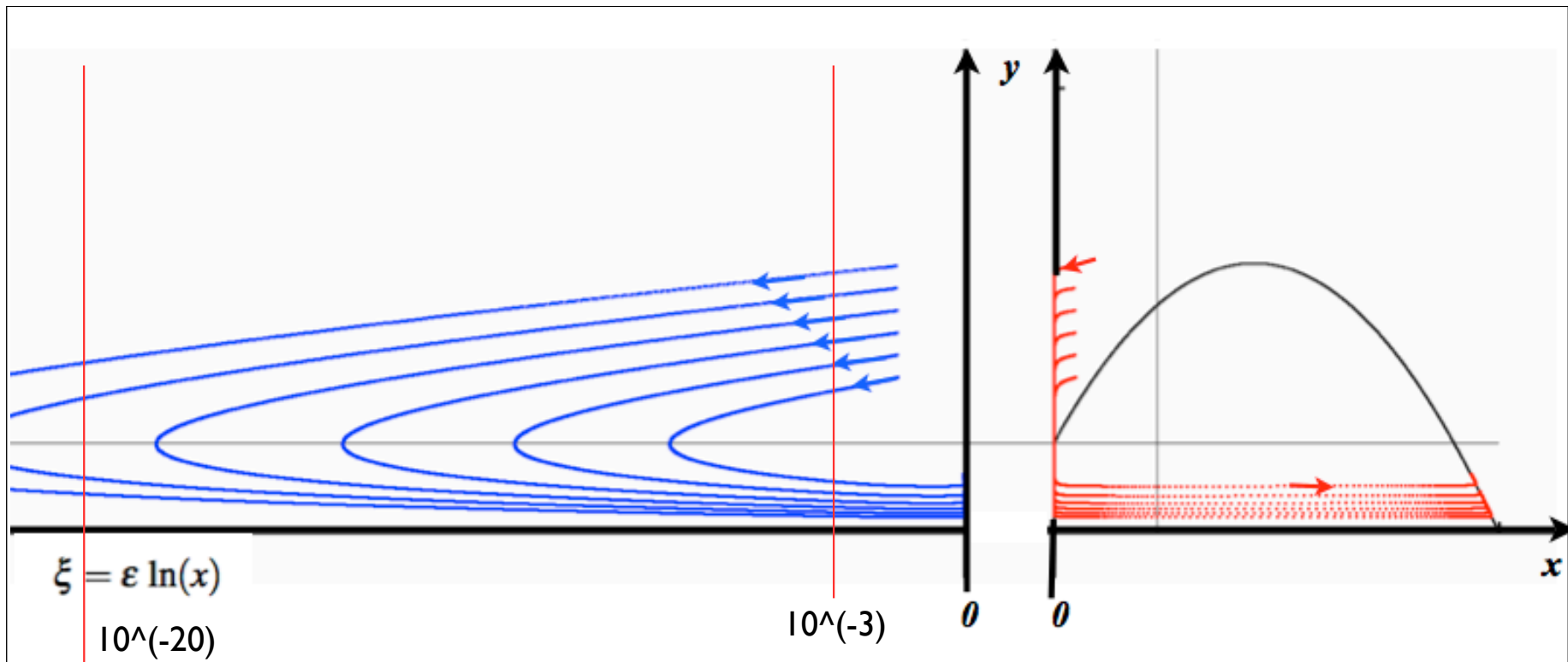


FIG. 4 – Quelques trajectoires en variable ξ avec $\varepsilon = 0.05$ et $m = 0.7$.

$$\begin{cases} \frac{d\xi}{dt} = 2 - 10y \\ \frac{dy}{dt} = -my \end{cases}$$

$$\begin{cases} \frac{dx}{dt} = \frac{1}{\varepsilon} \left(2x(1-x) - \frac{xy}{0.1+x} \right) \\ \frac{dy}{dt} = \left(\frac{x}{0.1+x} - m \right) y \end{cases}$$

Le même avec du bruit

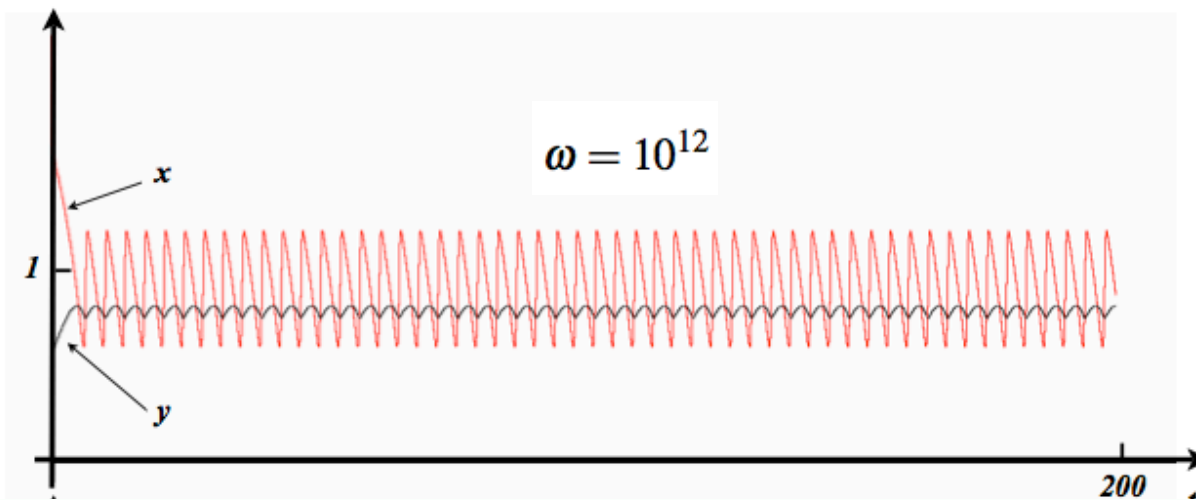
$$\begin{cases} \frac{dx}{dt} = \frac{1}{\varepsilon} [f(x) - \mu(x)y] \\ \frac{dy}{dt} = (\mu(x) - m)y \end{cases} \quad \begin{array}{l} - f(x) = \frac{1}{2}x(2-x) \\ - \mu(x) = \frac{x}{0.4+x} \\ - \varepsilon = 0.02 \end{array}$$

$$x(t+dt) - x(t) \approx dt \frac{1}{\varepsilon} [f(x(t)) - \mu(x(t))y(t)] - \sqrt{dt \frac{4}{\omega\varepsilon} \frac{f(x(t))\mu(x(t))y(t)}{(f(x(t)) + \mu(x(t))y(t))}} W_t$$

$$y(t+dt) - y(t) \approx dt [\mu(x(t)) - m]y(t) + \sqrt{dt \frac{\varepsilon}{\omega} \frac{f(x(t))\mu(x(t))y(t)}{(f(x(t)) + \mu(x(t))y(t))}} W_t$$

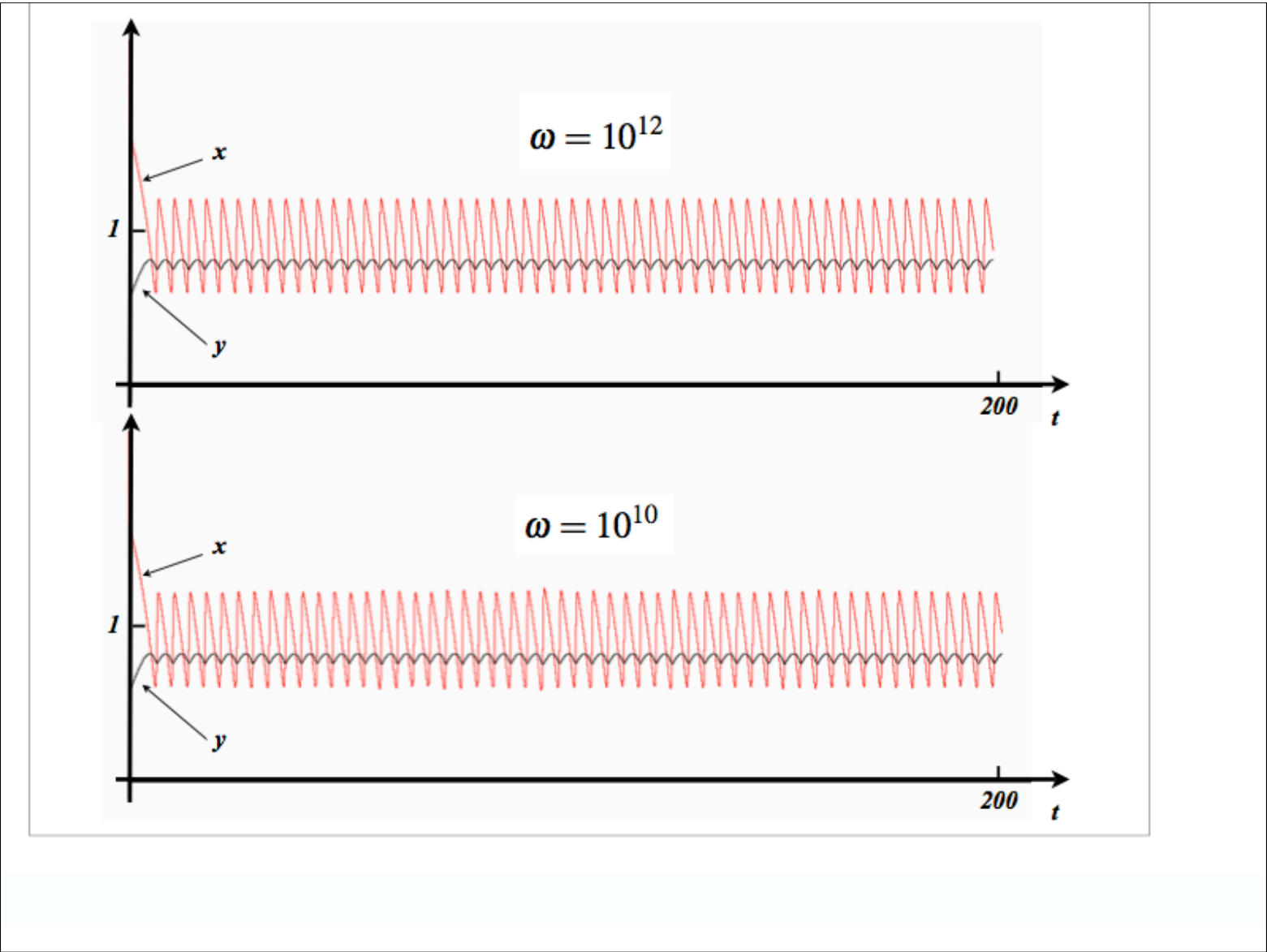
1 unit of $x = \omega$ individuals

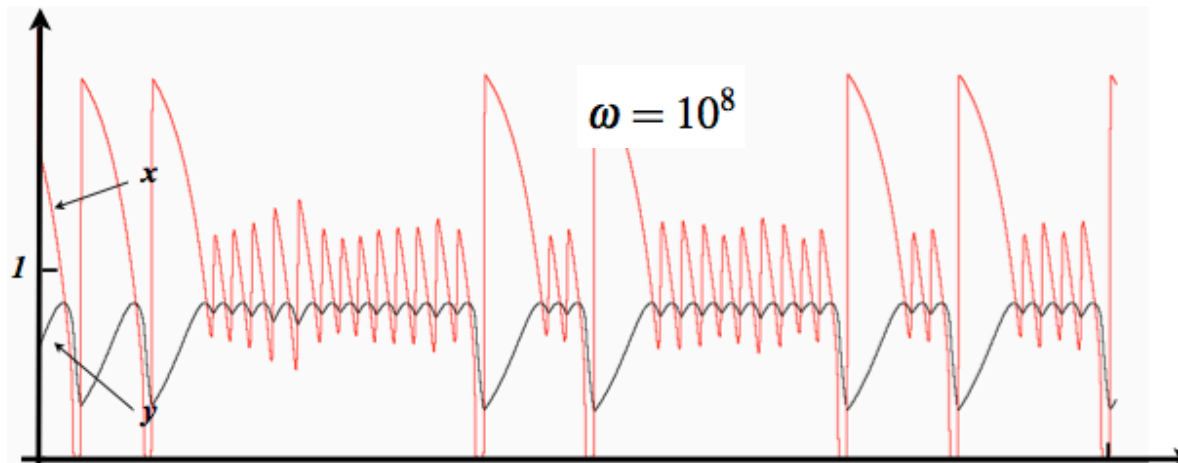
La théorie des processus de vie et mort permet de justifier le fait que si une unité de biomasse représente "omega" individus alors la variance du bruit doit avoir cette forme.



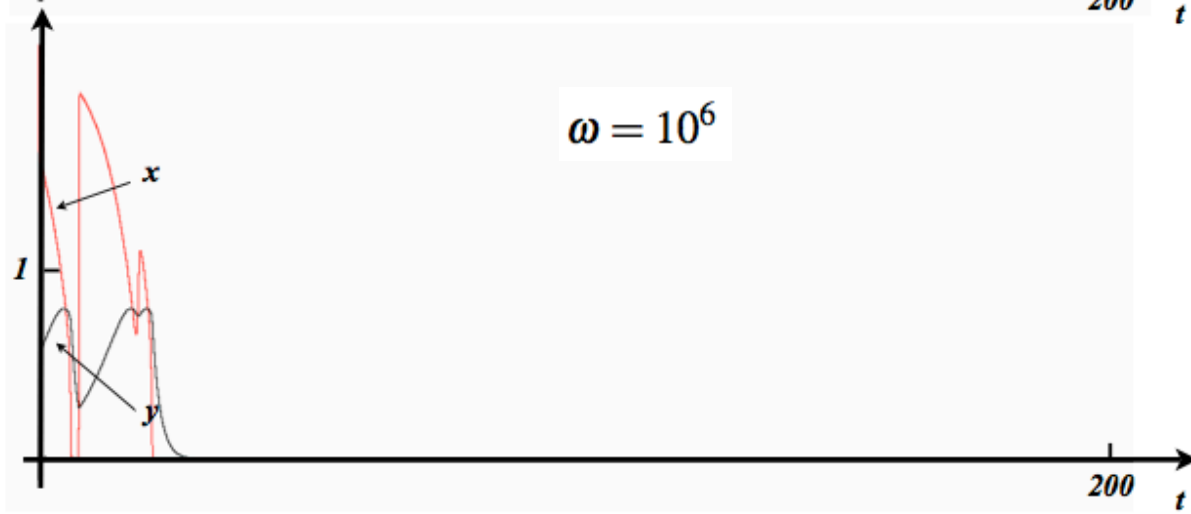
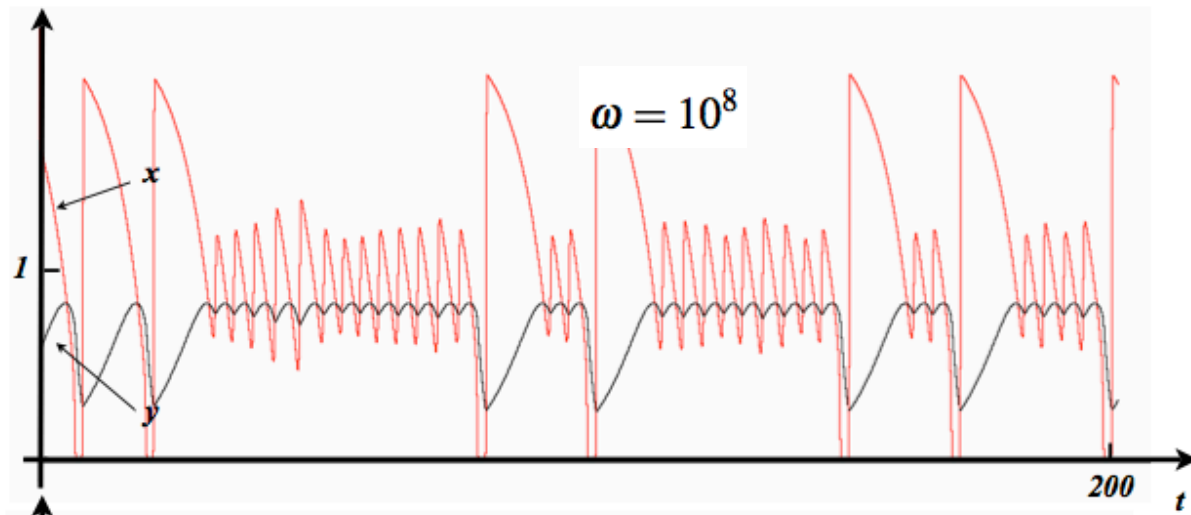
Not a surprise

$$\begin{cases} \frac{dx}{dt} = \frac{1}{\varepsilon} [f(x) - \mu(x)y] \\ \frac{dy}{dt} = (\mu(x) - m)y \end{cases} \quad + \text{ very (very) small noise}$$





This is a (big) surprise since the number of individuals is still very large



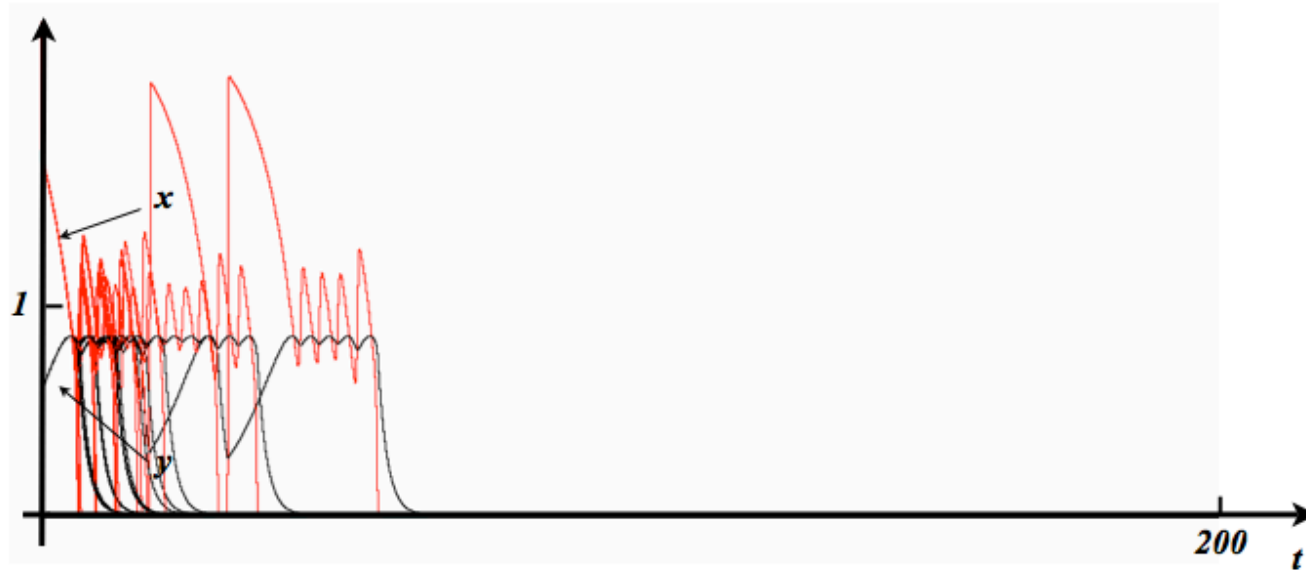


FIG. 3 – Twenty runs with $\omega = 10^6$

$$x(t+dt) - x(t) \approx dt \frac{1}{\varepsilon} [f(x(t)) - \mu(x(t))y(t)] - \sqrt{dt \frac{4}{\omega \varepsilon} \frac{f(x(t))\mu(x(t))y(t)}{(f(x(t)) + \mu(x(t))y(t))}} W_t$$

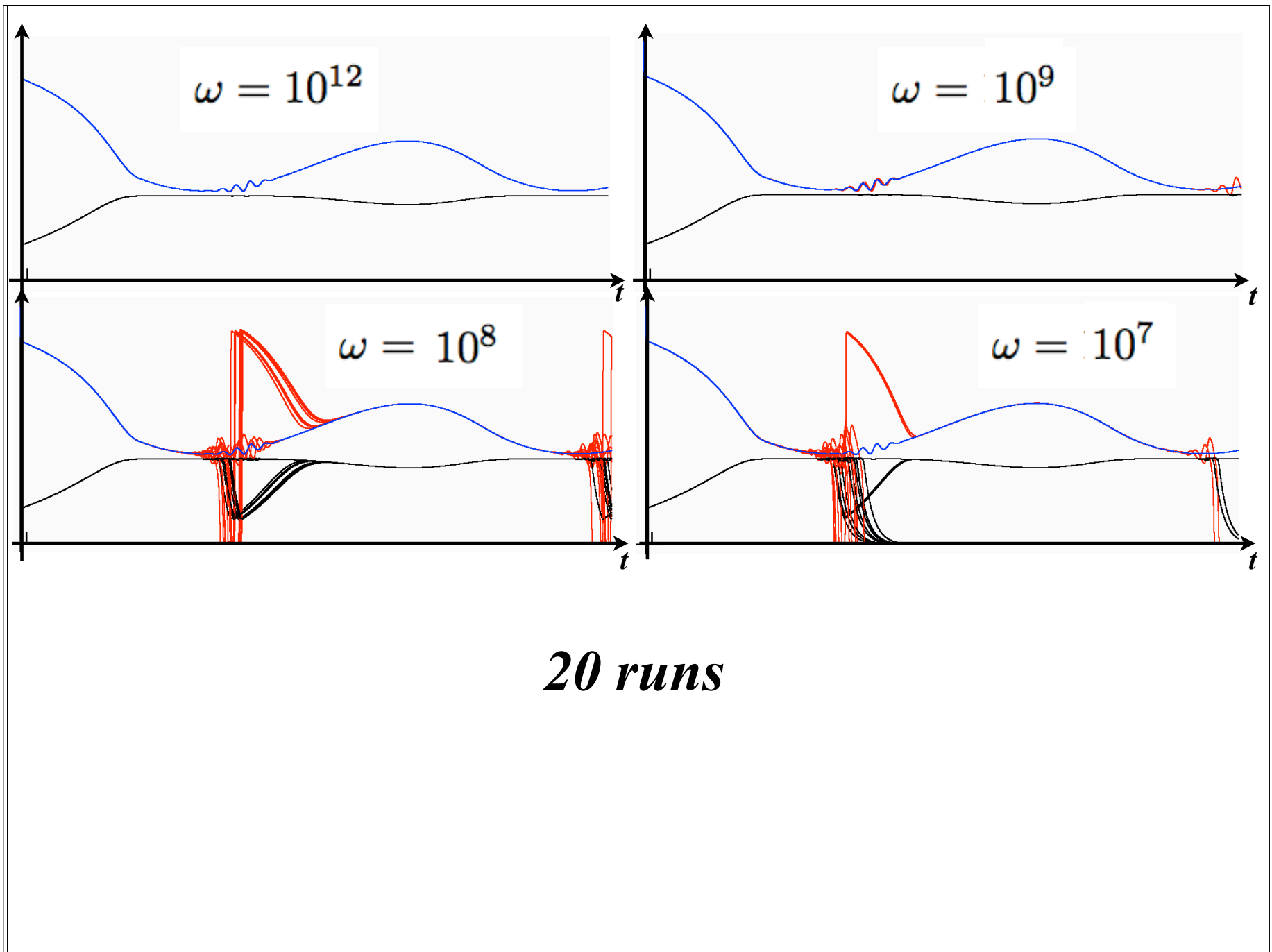
$$y(t+dt) - y(t) \approx dt [\mu(x(t)) - m]y(t) + \sqrt{dt \frac{\varepsilon}{\omega} \frac{f(x(t))\mu(x(t))y(t)}{(f(x(t)) + \mu(x(t))y(t))}} W_t$$



$$m(t) = a + b \cos(r t)$$

Simulations : In blue = deterministic part only

Simulations : In red = full model





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Effect of population size in a predator–prey model

F. Campillo*, C. Lobry

Modemic Team-Project INRA/INRIA, SupAgro, 2 Place Viala, 34060 Montpellier Cedex 2, France

I must confess that, at least for me, these results are hard to believe. I agree that, quite often, deterministic models are interpreted with unacceptably low populations (e.g., a few dozens individuals). But 10^8 is a huge number, far exceeding any real population, except in microbiology. The first question that comes to mind is: aren't these results numerical artifacts? Reading through the paper, I couldn't find a discussion about this.

Les deux commentaires de
referee qui suivent sont
intéressants



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Effect of population size in a predator–prey model

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This paper is quite interesting. It couples a stochastic model with a differential equation model to describe a predator-prey interacting dynamic system. The major result is that a continuous representation of the population abundance appears to be valid only above 10^8 individuals. If this is true, then almost all continuous models of population dynamics should be thrown away.

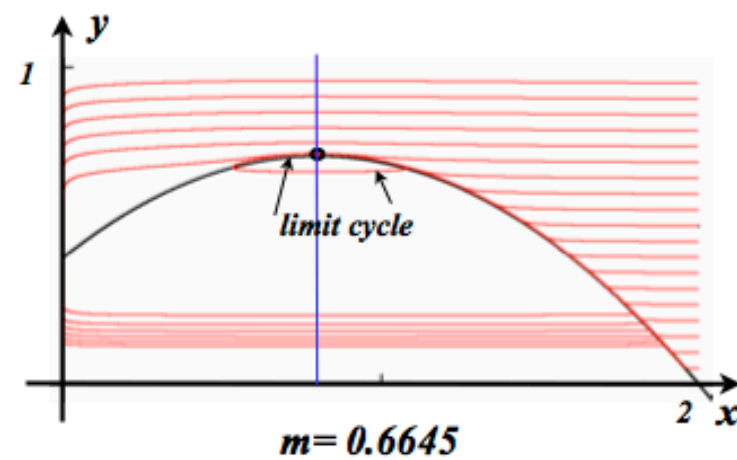
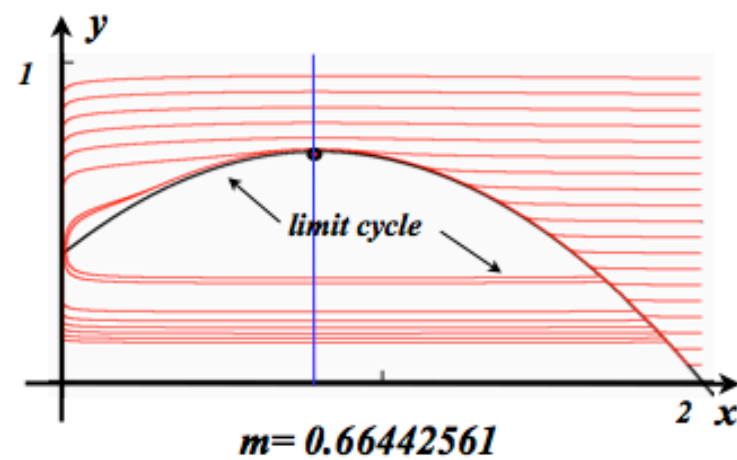
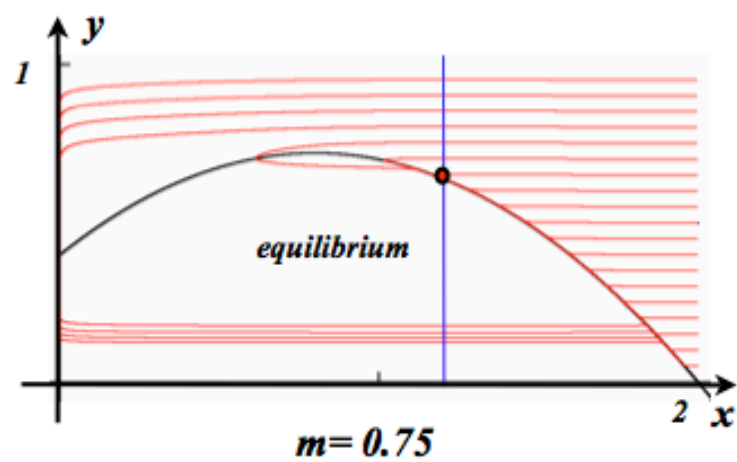
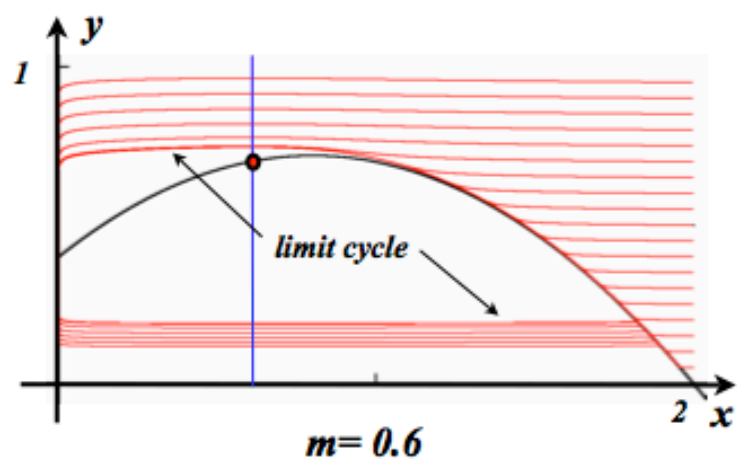


FIG. 6 – Phase portrait of system (7) for different values of m

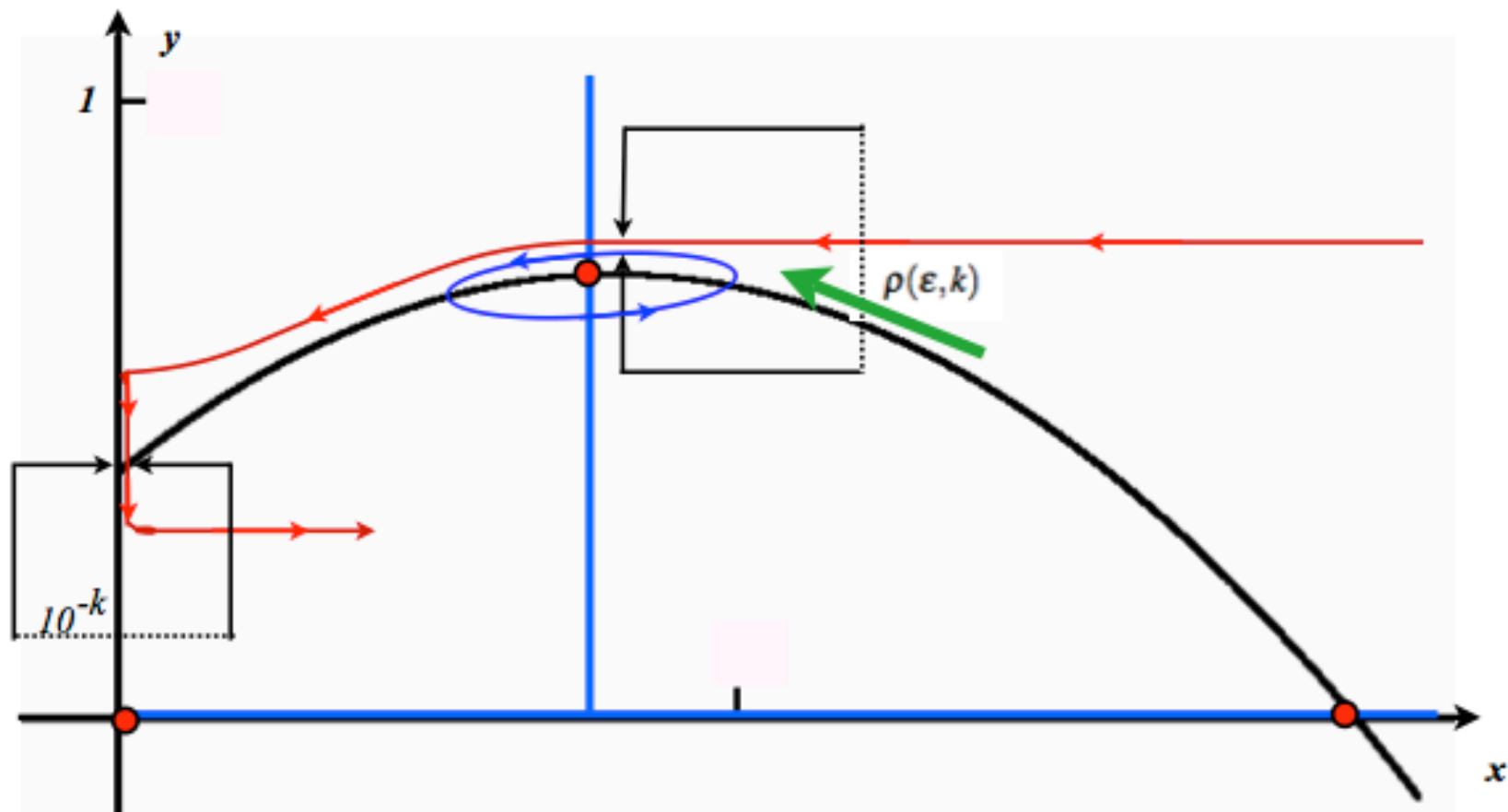
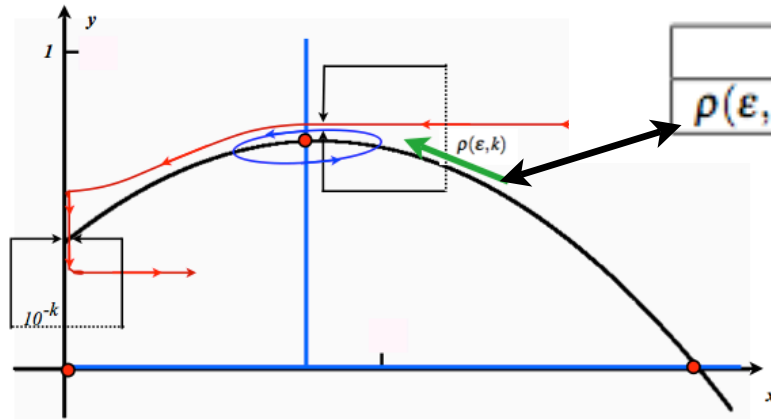


FIG. 9 – The “safety funnel”

ω	10^9	10^8	10^7	10^6
$\rho(\epsilon, k)$	0.001208	0.000114	0.000054	0.000052



ω	10^9	10^8	10^7	10^6
$\rho(\epsilon, k)$	0.001208	0.000114	0.000054	0.000052

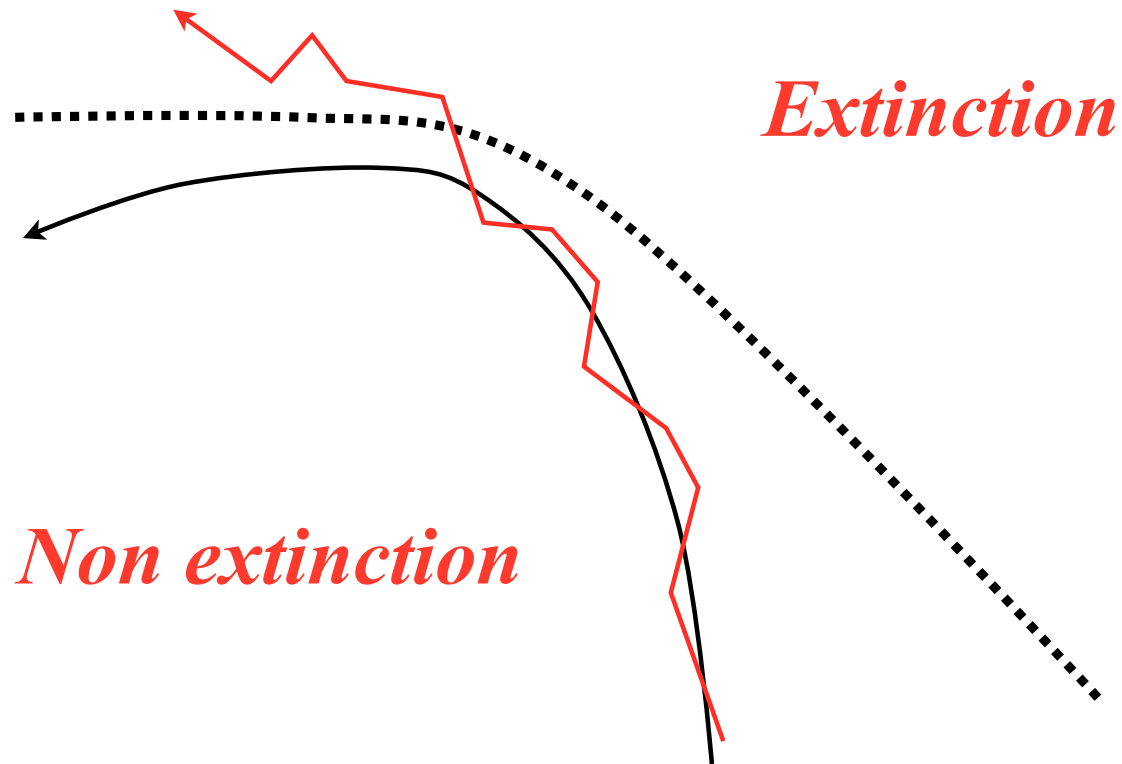
FIG. 9 – The “safety funnel”

$$x(t+dt) - x(t) \approx dt \frac{1}{\epsilon} [f(x(t)) - \mu(x(t))y(t)] - \sqrt{dt \frac{4}{\omega \epsilon} \frac{f(x(t))\mu(x(t))y(t)}{(f(x(t)) + \mu(x(t))y(t))}} W_t$$

$$y(t+dt) - y(t) \approx dt [\mu(x(t)) - m]y(t) + \sqrt{dt \frac{\epsilon}{\omega} \frac{f(x(t))\mu(x(t))y(t)}{(f(x(t)) + \mu(x(t))y(t))}} W_t$$

En théorie des perturbations singulières la théorie dite des “canard” explique parfaitement pourquoi cet entonnoir exponentiellement étroit est inéluctable.

$y(t)$



Non extinction

Extinction

$x(t)$

Le problème “atto - fox” n’est pas qu’un problème de “petite population”. En raison du bruit démographique inéluctable il faut se méfier des modèles où l’on passe trop près de la frontière des bassins d’attraction.

Bibliographie



Graphical Representation and Stability Conditions of Predator-Prey Interactions

M. L. Rosenzweig; R. H. MacArthur

American Naturalist, Vol. 97, No. 895 (Jul. - Aug., 1963), 209-223.

Stable URL:

<http://links.jstor.org/sici?sici=0003-0147%28196307%2F08%2997%3A895%3C209%3AGRASCO%3E2.0.CO%3B2-L>

Now we connect all the points at which the prey population just maintained itself, and call the resulting line the prey isocline $\left(\frac{d \text{ Prey}}{dt} = 0 \text{ for all points on the line} \right)$. Let us also connect the points at which the predator population just maintained itself and call this the predator isocline. If the lines intersect, neither population will be changing at the point of intersection. Such points, if any exist, are called the equilibrium points of the interaction.

Bibliographie

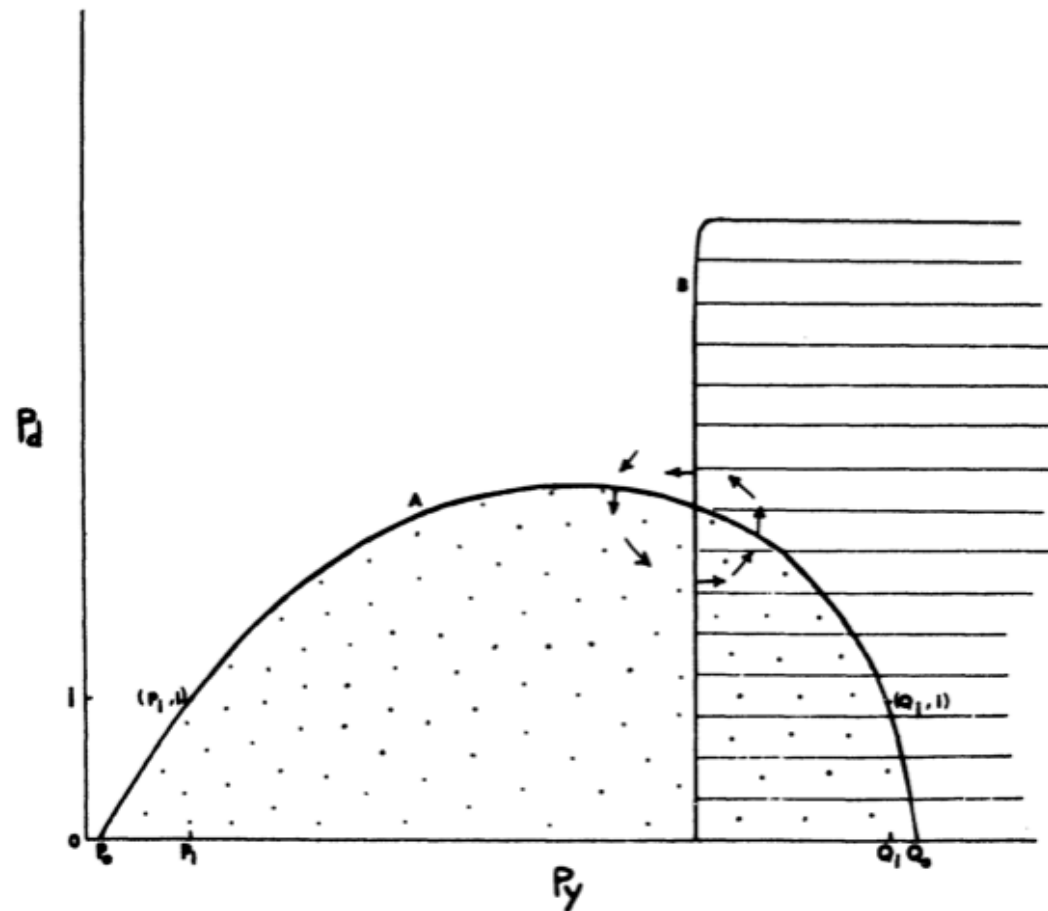


FIGURE 1. The instantaneous model of the interaction of a food-limited predator and its prey. P_y = prey density; P_d = predator density. Line A is the prey isocline, that is, the set of all points for which $\frac{dP_y}{dt} = 0$; line B is the predator isocline. P_y increases in the dotted area only; P_d increases only in the shaded area. The vectors are the instantaneous (general) direction of change of the community at eight qualitatively-different points in the graph.

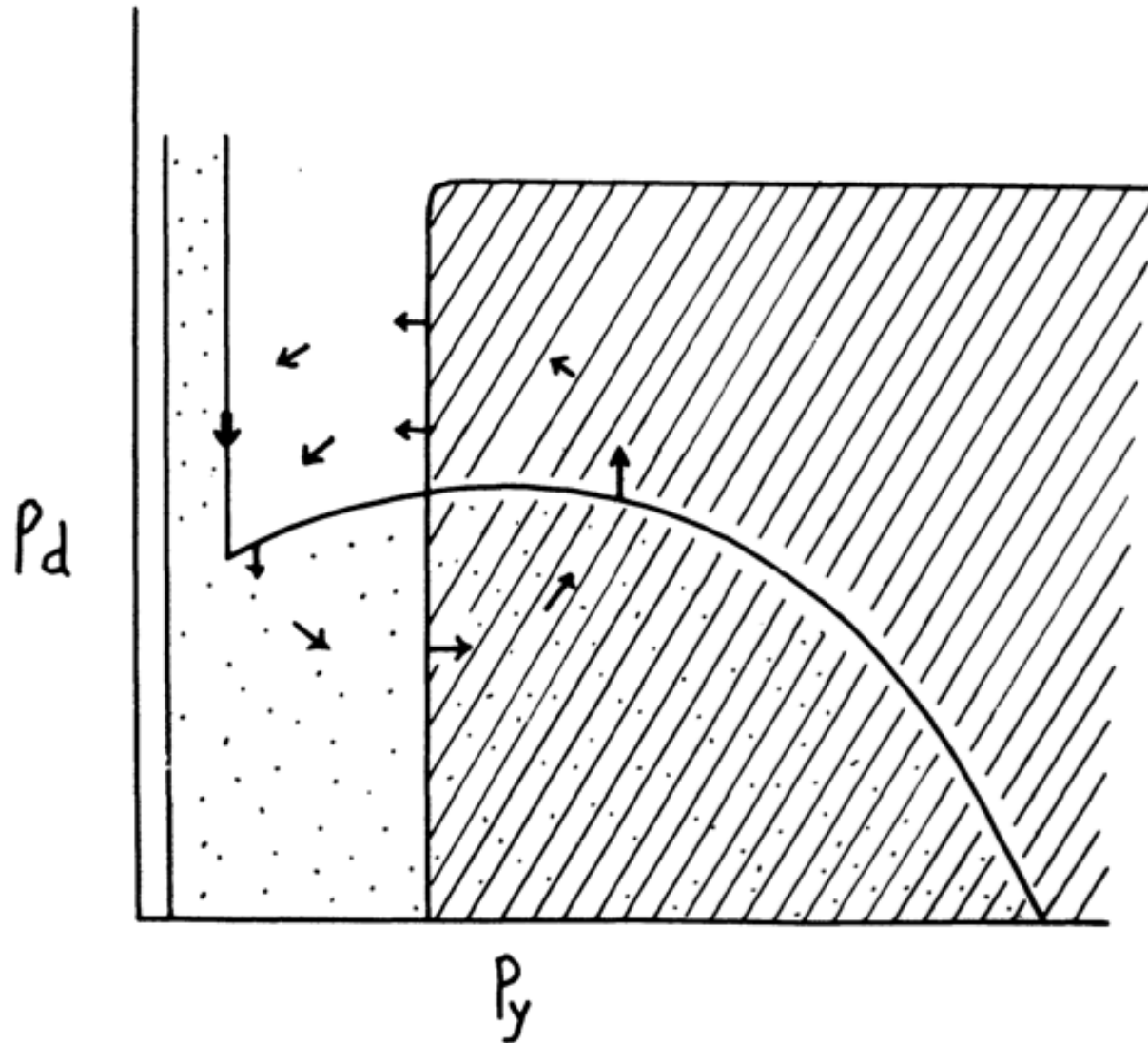


FIGURE 5. The interaction of a predator and its prey where the predator is unable to eat the prey at every prey density. Isoclines, coordinates, and markings as in figure 1. In this case, growing oscillations do not necessarily grow until one or both species becomes extinct, for a limiting oscillation may be reached.

Bibliographie

The predator isocline, $\frac{dPd}{dt} = 0$, is simpler to deduce. The predator is depending on the prey for its ability to increase. When the prey fall below a certain level, the predators decrease; when they are greater than this level, the predators increase. Since we are dealing with an instantaneous model, the predators will be increasing if and only if they are eating prey at the required rate (Andrewartha and Browning, 1961). Attainment of this rate depends only on the prey density. It is true that the greater the number of predators, the faster the density is reduced; still the instantaneous rate of change of the predator population depends only on the instantaneous rate of kill, which depends on the instantaneous density of prey. This re-

Bibliographie

LITERATURE CITED

- Allen, Durward L., 1962, Our wildlife legacy. Rev. ed. Funk and Wagnalls, New York. 422 pp.
- Andrewartha, H. G., and T. O. Browning, 1961, An analysis of the idea of "resources" in animal ecology. *J. Theoret. Biol.*, 1: 83-97.
- Christian, J. J., 1961, Phenomena associated with population density. *Proc. Nat. Acad. Sci. U. S.*, 47: 428.
- Connell, J. H., 1961, Effects of competition, predation by *Thais lapillus*, and other factors on natural populations of the barnacle, *Balanus balanoides*. *Ecol. Monographs*, 31: 61-104.
- Crowell, K., 1961, Effects of reduced competition in birds. *Proc. Nat. Acad. Sci. U. S.*, 47(2): 240-3.
- Holling, C. S., 1959, The components of predation as revealed by a study of small-mammal predation of the European sawfly. *Can. Entomol.*, 91(5): 293-320.
- Huffaker, C. B., 1958, Experimental studies on predation: dispersion factors and predator-prey oscillations. *Hilgardia*, 27: 343-383.
- Lotka, A. J., 1932, The growth of mixed populations; two species competing for a common food supply. *J. Wash. Acad. Sci.*, 22: 461-469.
- MacArthur, R. H., 1960, On the relation between reproductive value and optimal predation. *Proc. Nat. Acad. Sci. U. S.*, 46(1): 143-145.
- Slobodkin, L. B., 1961, Growth and regulation of animal populations. Holt, Rinehart, and Winston, New York. 184 pp.
- Utida, S., 1957, Population fluctuation, an experimental and theoretical approach. *Cold Spring Harbor Symp. Quant. Biol.*, 22: 139-151.

Bibliographie

LITERATURE CITED

- Allen, Durward L., 1962, Our wildlife legacy. Rev. ed. Funk and Wagnalls, New York. 422 pp.
- Andrewartha, H. G., and T. O. Browning, 1961, An analysis of the idea of "resources" in animal ecology. *J. Theoret. Biol.*, 1: 83-97.
- Christian, J. J., 1961, Phenomena associated with population density. *Proc. Nat. Acad. Sci. U. S.* 47: 428.

Manque :

- Gause

- Volterra

- Kolmogorov

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- Lotka, A. J., 1932, The growth of mixed populations; two species competing for a common food supply. *J. Wash. Acad. Sci.*, 22: 461-469.
- MacArthur, R. H., 1960, On the relation between reproductive value and optimal predation. *Proc. Nat. Acad. Sci. U. S.*, 46(1): 143-145.
- Slobodkin, L. B., 1961, Growth and regulation of animal populations. Holt, Rinehart, and Winston, New York. 184 pp.
- Utida, S., 1957, Population fluctuation, an experimental and theoretical approach. *Cold Spring Harbor Symp. Quant. Biol.*, 22: 139-151.

: *lapillus*,
, *Balanus*

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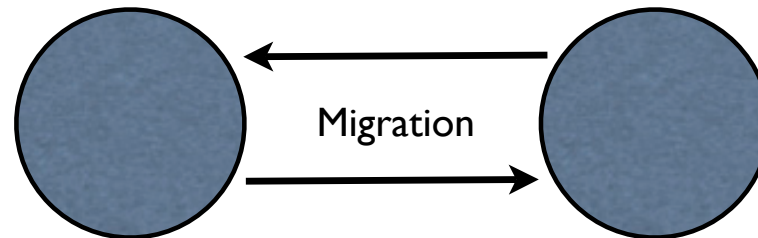
Migrations in the Rosenzweig-MacArthur model and the “atto-fox” problem

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ARIMA



$$\left\{ \begin{array}{l} \frac{dx_1}{dt} = \frac{1}{\varepsilon} (x_1 g(x_1) - h(x_1) y_1) + \nu_x(x_2, y_2) - \nu_x(x_1, y_1) \\ \frac{dy_1}{dt} = (h(x_1) - m) y_1 + \nu_y(x_2, y_2) - \nu_y(x_1, y_1) \\ \frac{dx_2}{dt} = \frac{1}{\varepsilon} (x_2 g(x_2) - h(x_2) y_2) + \nu_x(x_1, y_1) - \nu_x(x_2, y_2) \\ \frac{dy_2}{dt} = (h(x_2) - m) y_2 + \nu_y(x_1, y_1) - \nu_y(x_2, y_2) \end{array} \right. \quad (10)$$