# Inverse boundary value problems for elliptic PDEs and best approximation issues in classes of analytic functions

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The strong links between harmonic functions in domains $\Omega$ of $\mathbb{R}^2 \simeq \mathbb{C}$ and analytic functions of the complex variable allow to handle quantity of issues related to Laplace equation by using tools from complex and functional analysis, among which best constrained approximation in normed Hardy spaces. These issues include direct and inverse boundary problems, namely Cauchy transmission problems and coefficients or geometry identification issues [5, 7]. Computational algorithms are provided by solving the extremal problems. Stability properties together with error estimates and robustness are also handled [4].
We will discuss the following bounded extremal problems in Hardy Hilbert spaces $H^2(\Omega)$ of functions analytic in $\Omega$ and whose trace are bounded in $L^2(\partial\Omega)$ norm, whenever $\Omega$ is (conformally equivalent to) a disc or an annulus [2]. Let $I \subset \partial\Omega$ be a proper subset of $\partial\Omega$ and its complementary $J = \partial\Omega \setminus I$, both with positive Lebesgue measure.
Being given $h \in L^2(J)$, and $M > 0$, let

$$B = \{g \in H^2(\Omega)\,,\ \|h - g\|_{L^2(J)} \leq M\}_{|_I}\,,$$

be the approximation class. For $f \in L^2(I)$, we look for a solution $g_* \in B$ to the bounded extremal problem:

$$\min_{g \in B} \|f - g\|_{L^2(I)} = \|f - g_*\|_{L^2(I)}\,.$$

Existence and uniqueness properties will be described, together with constructive aspects that involve Toeplitz operators (with symbol the indicator function of $J$).

As applications, we will consider and solve some inverse problems for harmonic functions in planar domains, related to physical or engineering issues [3].

Extensions to other elliptic partial differential equations and generalized analytic functions, and to higher dimensional situations (in classes of gradients of harmonic functions) will be briefly presented [1, 6].

## References


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