Accurate numerical methods for solving exploding systems of differential equations

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Abstract

This paper considers the mathematical framework for computation of explosive solutions to ordinary and partial differential equations. The method used generates automatically a sequence of non uniform slices $\{[T_{n-1}, T_n]|n \geq 1\}$ determined by an end-of-slice condition that controls the growth of the solution within each slice. It also uses rescaling of the variables, time and variable(s) describing the solution. Thus, the original system is transformed into a sequence of slices-dependent initial-value shooting problems. Suitable selection of this change of variables leads the rescaled systems to satisfy a concept of **uniformity**, allowing to disable the extreme stiffness of the original problem. The uniformly rescaled systems are locally solved using standard solvers, within a computational tolerance of ϵ_{loc}. The sequential implementation of the local solver on a total of N slices leads to approximating the original solution of the system within a global tolerance ϵ_{glob}. We derive a relationship between ϵ_{loc}, ϵ_{glob} and N. Assessment of these estimates is done through numerical experiments that are conducted for infinite and finite times explosive discrete reaction diffusion problems.

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